# Risk Minimization in Power System Expansion and Power Pool Electricity Markets

by

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I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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# Abstract

Centralized power system planning covers time windows that range from ten to thirty years. Consequently, it is the longest and most uncertain part of power system economics. One of the challenges that power system planning faces is the inability to accurately predict random events; these random events introduce risk in the planning process. Another challenge stems from the fact that, despite having a centralized planning scheme, generation plans are set first and then transmission expansion plans are carried out. This thesis addresses these problems. A joint model for generation and transmission expansion for the vertically integrated industry is proposed. Randomness is considered in demand, equivalent availability factors of the generators, and transmission capacity factors of the transmission lines. The system expansion model is formulated as a two-stage stochastic program with fixed recourse and probabilistic constraints. The transmission network is included via a DC approximation. The mean variance Markowitz theory is used as a risk minimization technique in order to minimize the variance of the annualized estimated generating cost. This system expansion model is capable of considering the locations of new generation and transmission and also of choosing the right mixture of generating technologies.

The global tendency is to move from regulated power systems to deregulated power systems. Power pool electricity markets, assuming that the independent system operator is concerned with the social cost minimization, face great uncertainties from supply and demand bids submitted by market participants. In power pool electricity markets, randomness in the cost and benefit functions through random demand and supply functions has never been considered before. This thesis considers as random all the coefficients of the quadratic cost and benefit functions and uses the mean variance Markowitz theory to minimize the social cost variance. The impacts that this risk minimization technique has on nodal prices and on the elasticities of the supply and demand curves are studied.

All the mathematical models in this thesis are exemplified by the six-node network proposed by Garver in 1970, by the 21-node network proposed by the IEEE Reliability Test System Task Force in 1979, and by the IEEE 57- and 118-node systems.

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*To my beloved parents, Juan and Eva*

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# Nomenclature

**Acronyms:**

| | |
|---|---|
| AC | Alternating Current |
| DC | Direct Current |
| EEV | Expected Value Solution |
| EVPI | Expected Value of Perfect Information |
| FTR | Financial Transmission Right |
| GAMS | General Algebraic Modeling System |
| ISO | Independent System Operator |
| LDC | Load Duration Curve |
| LMP | Locational Marginal Price |
| SS | Stochastic Solution |
| VSS | Value of the Stochastic Solution |
| WSS | Wait and See Solution |

**Indices:**

| | |
|---|---|
| $et$ | Index for existing generating technologies. |
| $h, i, j, p, \nu$ | Indices for nodes in the system. |
| $i$-$j$ | Index for node pairs in the system. |
| $k$ | Index for foreseeable scenarios. |
| $m$ | Index for operating modes (load blocks). |

$nt$ Index for new generating technologies.

$s$ Index for the slack node.

$t$ Index for stages.

**Sets:**

$\mathcal{E}_t$ Set of existing generating technologies.

$\mathcal{H}$ Set of nodes with installed generating capacity.

$\mathcal{I}$ Set of node pairs with right-of-ways or lines constructed between them.

$\mathcal{K}$ Set of all foreseeable scenarios.

$\mathcal{M}$ Set of operating modes.

$\mathcal{N}$ Set of all different nodes.

$\mathcal{N}_t$ Set of new technologies.

$\mathcal{P}$ Set of nodes where new generating capacity can be installed.

$\mathcal{T}$ Set of stages.

**Functions, vectors and matrices:**

$\boldsymbol{d} \in \Re^{|\mathcal{N}|}$ Demand vector with elements $d_\nu$; in MW.

$\boldsymbol{d_k} \in \Re^{|\mathcal{N}|}$ Demand vector with elements $d_{k,\nu}$ for all $m$; in MW.

$\boldsymbol{d_m} \in \Re^{|\mathcal{N}|}$ Demand vector with elements $d_{m,\nu}$ for all $m$; in MW.

$\boldsymbol{d_{k,m}} \in \Re^{|\mathcal{N}|}$ Demand vector with elements $d_{k,m,\nu}$ for all $k, m$; in MW.

$\boldsymbol{d^r} \in \Re^{|\mathcal{N}|-1}$ Reduced demand vector with elements $d_\nu$; in MW.

$\boldsymbol{e} \in \Re^{|\mathcal{N}|-1}$ Unitary vector; dimensionless.

$\boldsymbol{f} \in \Re^{|\mathcal{I}|}$ Power-flow vector with elements $f_{i\text{-}j}$; in MW.

$\boldsymbol{f_k} \in \Re^{|\mathcal{I}|}$ Power-flow vector with elements $f_{k,i\text{-}j}$ for all $k$; in MW.

$\boldsymbol{f_m} \in \Re^{|\mathcal{I}|}$ Power-flow vector with elements $f_{m,i\text{-}j}$ for all $m$; in MW.

$\boldsymbol{f_{k,m}} \in \Re^{|\mathcal{I}|}$ Power-flow vector with elements $f_{k,m,i\text{-}j}$ for all $k, m$; in MW.

$\boldsymbol{g} \in \Re^{|\mathcal{N}|}$ Generation vector with elements $g_\nu = \sum_{et}^{et^+} g_{et,h}$; in MW.

$\boldsymbol{g^r} \in \Re^{|\mathcal{N}|-1}$ Reduced generation vector with elements $g_\nu$; in MW.

$\boldsymbol{g_k} \in \Re^{|\mathcal{N}|}$ Generation vector with elements $g_{k,\nu} = \sum_{et}^{et^+} g_{k,et,h}$ for all $k$; in MW.

$\boldsymbol{g_m} \in \Re^{|\mathcal{N}|}$ Generation vector with elements $g_{m,\nu} = \sum_{m,et}^{m^+,et^+} g_{m,et,h} + \sum_{m,nt}^{m^+,nt^+} g_{m,nt,p}$ for all $m$; in MW.

$\boldsymbol{g_{k,m}} \in \Re^{|\mathcal{N}|}$ Generation vector with elements $g_{k,m,\nu} = \sum_{m,et}^{m^+,et^+} g_{k,m,et,h} + \sum_{m,nt}^{m^+,nt^+} g_{k,m,nt,p}$ for all $k, m$; in MW.

$\boldsymbol{H} \in \Re^{|\mathcal{I}|\times|\mathcal{N}|-1}$ Transfer admittance matrix.

$\boldsymbol{S} \in \Re^{|\mathcal{N}|\times|\mathcal{I}|}$ Node-branch incidence matrix; dimensionless.

$\boldsymbol{\rho} \in \Re^{|\mathcal{N}|-1}$ Reduced price vector with elements $\rho_\nu$; in MW.

$\boldsymbol{\omega} \in \Re^{|\mathcal{N}|-1}$ Congestion price vector with elements $\omega_\nu$; in MW.

**Constants:**

$a_{1,s}$ Cost function intercept at $s$; in \$/MWh.

$a_{2,s}$ Cost function slope at $s$; in \$/MW$^2$h.

$a_{1,\nu}$ Cost function intercept at $\nu$; in \$/MWh.

$a_{2,\nu}$ Cost function slope at $\nu$; in \$/MW$^2$h.

$b_{1,s}$ Benefit function intercept at $s$; in \$/MWh.

$b_{2,s}$ Benefit function slope at $s$; in \$/MW$^2$h.

$b_{1,\nu}$ Benefit function intercept at $\nu$; in \$/MWh.

$b_{2,\nu}$ Benefit function slope at $\nu$; in \$/MW$^2$h.

$B^t$ Annual budget for each $t$; in \$/year.

$B$ Annual budget; in \$/year.

$c_{i\text{-}j}$ Annualized cost per circuit added in the right-of-way between $i$ and $j$; in \$/year.

$cf_m$ Capacity factor determined by $m$; dimensionless.

$cf_m^t$ Capacity factor determined by $m$ at $t$; dimensionless.

$d_\nu$ Demand at $\nu$; in MW.

$d_{m,\nu}$ Demand at $\nu$ for all $m$; in MW.

$d_{k,m,\nu}$ Demand at $\nu$ for all $k, m$; in MW.

$L_m^t$ Power demand in each $m$ at $t$; in MW.

$L_m$ Power demand in each $m$; in MW.

$n_{i\text{-}j}^0$ Number of existing circuits between $i$ and $j$; dimensionless.

$pr_k$ Probability of occurrence of $k$; dimensionless.

$P^t$ $P^t = \sum_m L_m^t$. Peak demand at $t$; in MW.

$P$ $P = \sum_m L_m$. Peak demand; in MW.

$q_{et}^t$ Annualized variable generating cost of $et$ at $t$; in \$/MW-year.

$q_{et}$ Annualized variable generating cost of $et$; in \$/MW-year.

$q_{nt}^t$ Annualized variable generating cost of $nt$ at $t$; in \$/MW-year.

$q_{nt}$ Annualized variable generating cost of $nt$; in \$/MW-year.

$r_{nt}^t$ Annualized fixed investment cost of $nt$ at $t$; in \$/MW-year.

$r_{nt}$ Annualized fixed investment cost of $nt$; in \$/MW-year.

$x_{nt}$ Typical capacity size of $nt$; in MW.

$y_{et}^t$ Total installed generating capacity of $et$ at $t$; in MW.

$y_{et}$ Total installed generating capacity of $et$; in MW.

$y_{et,h}$ Total installed generating capacity of $et$ at $h$; in MW.

$\alpha_{et}$ Equivalent availability of $et$; dimensionless.

$\alpha_{nt}$ Equivalent availability of $nt$; dimensionless.

$\beta_g$ Quantile of $\alpha_{et}$ and $\alpha_{nt}$; dimensionless.

$\beta_t$ Quantile of $\alpha_{i\text{-}j}$; dimensionless.

$\Gamma_{i\text{-}j}$ Susceptance of the line between $i$ and $j$ where the admittance $Y$ is given in Siemens.

$\theta_r$ Risk factor; dimensionless.

**Variables:**

$d_s$ Demand at the slack node; in MW.

$f_{i\text{-}j}$ Power-flow in line that connects $i$ and $j$; in MW.

$f_{k,i\text{-}j}$ Power-flow in line that connects $i$ and $j$ for all $k$; in MW.

$f_{m,i\text{-}j}$ Power-flow in line that connects $i$ and $j$ for all $m$; in MW.

$f_{k,m,i\text{-}j}$ Power-flow in line that connects $i$ and $j$ for all $k, m$; in MW.

$g_s$ Generation at the slack node; in MW.

$g^t_{m,nt}$ Capacity of $nt$ effectively used in each $m$ at $t$; in MW.

$g_{k,m,nt}$ Capacity of $nt$ effectively used in each $m$ for all $k$; in MW.

$g_{et,h}$ Capacity of $et$ effectively used at $h$; in MW.

$g_{m,et,h}$ Capacity of $et$ effectively used in each $m$ at $h$; in MW.

$g_{k,et,h}$ Capacity of $et$ effectively used at $h$ for all $k$; in MW.

$g_{k,m,et}$ Capacity of $et$ effectively used in each $m$ for all $k$; in MW.

$g_{k,m,et,h}$ Capacity of $et$ effectively used in each $m$ at $h$ for all $k$; in MW.

$g_{m,nt}$ Capacity of $nt$ effectively used in each $m$; in MW.

$g_{m,nt,p}$ Capacity of $nt$ effectively used in each $m$ at $p$; in MW.

$g_{k,m,nt,p}$ Capacity of $nt$ effectively used in each $m$ at $p$ for all $k$; in MW.

$g^t_{m,et}$ Capacity of $et$ effectively used in each $m$ at $t$; in MW.

$g_{m,et}$ Capacity of $et$ effectively used in each $m$ at $t$; in MW.

$n^t_{nt}$ Integer variable. Number of new generators of $nt$ installed at $t$; dimensionless.

$n_{nt}$ Integer variable. Number of new generators of $nt$; dimensionless.

$n_{nt,p}$ Integer variable. Number of new generators of $nt$ at $p$; dimensionless.

$n_{i\text{-}j}$ Integer variable. Number of circuits added in the right-of-way between $i$ and $j$; dimensionless.

$w_{nt}^{t}$ Total capacity of $nt$ made available at $t$; in MW.

$\delta_{m,\nu}$ Nodal angle at $\nu$ for all $m$; in radians.

$\delta_{k,\nu}$ Nodal angle at $\nu$ for all $k$; in radians.

$\delta_{k,m,\nu}$ Nodal angle at $\nu$ for all $k, m$; in radians.

$\delta_{\nu}$ Nodal angle at $\nu$; in radians.

$\rho_s$ Price at the slack node; in \$/MWh.

**Symbology:**

$(\overline{\cdot}), (\underline{\cdot})$ Maximum and minimum value for $(\cdot)$.

$|(\cdot)|$ Cardinality of the finite set $(\cdot)$.

$et^{+}, nt^{+}$ equals $|\mathcal{E}_t|, |\mathcal{N}_t|$, respectively.

$m^{+}$ equals $|\mathcal{M}| - (|\mathcal{M}| - m)$.

$E\{(\cdot)\}$ Expected value of $(\cdot)$; same units as $(\cdot)$.

$\sigma^2_{(\cdot)}$ Variance of $(\cdot)$; squared units of $(\cdot)$.

$\sigma_{(\cdot)}$ Standard deviation of $(\cdot)$; same units as $(\cdot)$.

$\sigma_{(\cdot),(\cdot)}$ Covariance between the random variables $(\cdot)$ and $(\cdot)$; units of $(\cdot) \times (\cdot)$.

# Chapter 1

# Introduction

Electric power systems, regardless whether they are under a centralized or a liberalized regime, are exposed to a number of variations from various random events. The general meaning of uncertainty is *the inability* to accurately predict random events [1]. This inability to predict such random events is the one that introduces risk into power system economics.

Power system operation can be divided into two major functions: power system dynamics and power system economics. Power system dynamics covers time windows that range from fractions of a second to many minutes. The three components of power system dynamics, and their time intervals in brackets, are: i) transient stability (cycles to ten seconds), ii) dynamic stability (one to ten seconds), and iii) long-term dynamics (seconds to minutes). Power system economics covers time windows ranging from several minutes to several years. The functions covered by power system economics are: i) corrective control actions (real time), ii) economic dispatch (five to ten minutes), iii) unit commitment (one day to one week), iv) hydrothermal coordination (short-range, one day to one week; long-range, one week to one year [2]), v) pumped storage scheduling (one week), vi) fuel purchases (months to years), vii) maintenance scheduling (one year), and viii) power system planning (ten to thirty years) [3]. Figure 1.1 shows the time frame of power system economics.

For the case of a vertically integrated industry—where the generation, transmission, and distribution sectors are monitored and operated by the utility's central control system—

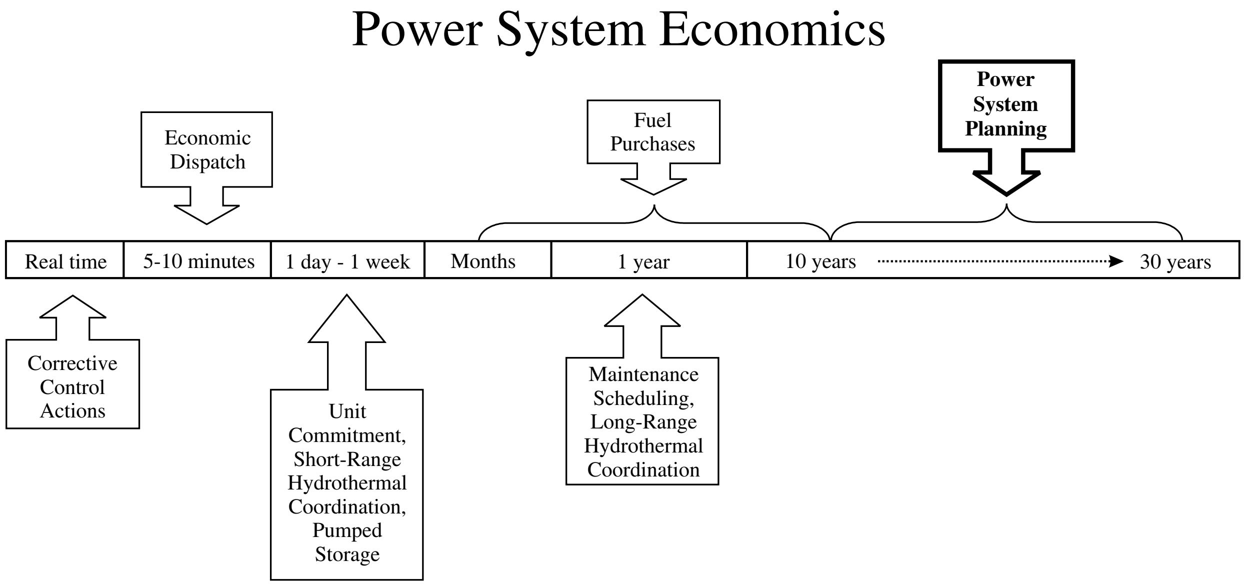


Figure 1.1: Functions covered by power system economics.

power system planning is the one that faces greater uncertainty. Since the planning for generation and transmission expansion ranges anywhere in between ten to thirty years, random events such as load growth, cost of capital, capital cost of new equipment, development of new technologies, cost of fuel, taxes, equivalent availabilities of the generating units, transmission capacity factors of the transmission lines, and even regulatory intervention introduce risk into the planning period [3].

For the case of a liberalized electricity industry—where the generation, transmission, and distribution sectors are owned and operated independently—the pool electricity market faces uncertainty from the suppliers' and consumers' bids submitted to the Independent System Operator (ISO).

Acknowledging the presence of risk is not enough; mathematical models that minimize risk are needed. To incorporate the effects of these sources of uncertainty into an optimization model, one can use a combination of random variables, stochastic programming, and probabilistic constraints. To minimize the effects of the risk caused by uncertainty, one can use any risk minimization technique such as the minimum variance approach devised by H. M. Markowitz in 1952 and widely used in financial engineering.

## 1.1 Centralized Power System Planning

Planning decisions made in the past affect the performance of power systems now, and planning decisions taken now will affect the performance of power systems in the future. Since power systems are expected to keep on growing as time goes, mainly because of technological advancement and population growth, power system planning is a key factor in long-term power system economics. Power system planning has to be done in such a way that the reliability of the system is not compromised. When to invest, how much capacity to add, what type of generation is needed, and where to locate new transmission lines and generating units are important decisions in power system planning [4]. In any decision making process, there are multiple choices which, in turn, produce multiple outcomes. Decisions can be made under three different situations: i) under certainty,[1] ii) under uncertainty,[2] and iii) under risk[3] [5, 6].

Power system planning is a multiple attribute decision making problem and, due to its long-term nature, all the investments made are irreversible [7]. The regulated electricity industry is concerned with the simultaneous minimization of various criteria such as the present worth of all future costs (including operating cost); cash flow deficiencies; environmental impacts on air, water, and land; probability of not meeting the forecasted demand; and social impacts of new construction. The challenge is that, most of the time, these different minimization criteria are inconsistent, *i.e.*, a plan that minimizes one does not minimize any of the others [3].

In the regulated electricity industry, generation and transmission expansion are carried out by the same agent. Motivated by quantity, not by price, and based on forecasted levels of demand, the system planner chooses the optimal investment levels of various types of power plants. After a generation plan is set, transmission expansion is carried out with the advantage of knowing the opening and closing of generating stations. Hence, investments in power system expansion are sequential in practice [8,9]. Usually, a constrained optimization approach is chosen in order to minimize the investment cost while avoiding load shedding

[1] No randomness, future is assumed to be fully and perfectly known.

[2] Multiple foreseeable scenarios; their probabilities are known or estimated over a particular period of time. Use of random variables.

[3] Risk minimization is incorporated.

and maintaining system reliability.

There are two types of transmission investments. The first type of network investment comes from enhancement/maintenance decisions on the existing network; these investments may include the addition, replacement, or upgrade of elements like: i) relays and switches, ii) remote monitoring and control equipment, iii) transformers, iv) substation facilities, v) capacitor banks, vi) conductors on existing links, and vii) voltage levels on transmission links. The second type of network investment, the one this work is concerned with, involves the building of new transmission lines in either existent or new rights-of-way [10].

The first step in generation and transmission expansion planning is to evaluate as many alternatives as possible. Some of these are the various possible configurations of new transmission lines and/or generating plants. To do this at a reasonable computational expense, the modeling has to be simplified. As the planning solution narrows just to a few possibilities, the modeling accuracy can be increased, *i.e.*, by including AC power flows, losses, stability studies, transient analysis, line design studies, and so forth [4]. The models presented in this work are intended to narrow the planning solutions.

## 1.2 Pool Electricity Markets

Since the appearance of the first decentralized power systems, Chile in 1982 and England in 1991, much effort has been focused on the short-term period of power system economics and on how to encourage competition among market participants. Most of the work has been done under the strong assumption of having a perfectly known future. However, in real life, many of the events involved in the market clearing process are random. For instance, unpredictable changes in fuel prices can affect the variable generating cost of suppliers, and changes in the economy can affect the price responsiveness of the demand. These random events not only influence the random variables in the model—like power flows, generation, and demand levels—but also the Locational Marginal Prices (LMPs). The inability to accurately predict these random variations introduce risk into the market clearing process. The greatest source of uncertainty, or risk, in a pool electricity market comes from the coefficients of the quadratic cost and benefit functions.

## 1.3 Research Motivation

Most of the work that has been done in generation and transmission expansion for a vertically integrated industry has been done in a deterministic way. Although the majority of the referenced work acknowledges the stochastic nature of the generation and transmission expansion problem, none has tried to develop a joint stochastic model for generation and transmission expansion. Stochastic models have considerable advantages over deterministic ones. One of these advantages is that the overall cost, investment and operational costs, is lower than the overall cost of its deterministic counterpart and that the stochastic model can meet the requirements for all the foreseeable scenarios, something that a deterministic model cannot do.

Developing stochastic models is not enough. As stated before, the inability to perfectly predict random events introduce risk into the planning process. Hence, an optimization model that minimizes the risk from these random events is needed. The mean-variance Markowitz theory can easily be applied; by means of a single risk factor in the objective function, one can explicitly account for the trade-offs between the mean and the variance.

For the case of pool electricity markets, it is of paramount importance to develop stochastic models. Since LMPs are one of the main components in computing important parameters used in deregulated power system planning, it is needed to explore the effects that risk minimization techniques have on them. It is also important to explore how the elasticities of the supply and demand curves are affected by one's position toward risk.

For the decentralized generation expansion case, profit-maximizing market participants invest in new generation based on their expectation of energy prices, return on new investments, and demand growth. Therefore, generation expansion is motivated by price; the revenues, based on LMPs, have to cover the capital and operating costs [11, 12]. When load faces real-time prices and is elastic enough, high energy prices indicate high levels of demand, and tight competitive supply and demand conditions. This, in turn, provides an incentive to invest in new generation [13]. Hence, a more accurate computation of LMPs that accounts for the uncertainty present in the supply and demand curves is needed.

For the decentralized transmission expansion case, there are numerous proposals in the literature about how one should provide incentives to invest in new transmission. Some of these are spatial variations of LMPs [14, 15], Financial Transmission Rights (FTRs) [10, 16, 17], and congestion costs [18] – [20]. If spatial variations of LMPs or FTRs are to be used as an incentive for decentralized transmission expansion, then again it would be better if the LMPs were computed taking into account the minimization of the risk introduced by the random supply and demand curves. For the case of congestion costs, since they are computed based on the assumption that the supply curves represent the true marginal costs, and that the demand curves represent the true willingness to pay, once more it would be better if the randomness in their parameters is taken into account.

## 1.4 Structure of the Thesis

This thesis is organized in five chapters and three appendices.

Chapter 2 takes a deterministic approach on how centralized power system planning is carried out. All the foundations are laid in order to formulate the generation and transmission expansion problem as an optimization model. A new model for joint generation and transmission expansion is also presented.

In Chapter 3, a stochastic approach on centralized power system planning is taken. Random events such as demand levels, equivalent availabilities of generating units, and transmission capacity factors of transmission lines are considered. The mean-variance Markowitz theory is introduced as a risk minimization technique. The use of probabilistic constraints is also explained to account for randomness on the coefficients of some of the constraints. Three new models are presented; one for generation expansion, one for transmission expansion, and one for joint generation and transmission expansion. These models are mixed-integer nonlinear optimization programs.

Chapter 4 opens with some comments on decentralized power system planning and defines some economic terms used in this chapter. Afterwards, the mean-variance Markowitz theory is applied to the clearing process of a pool electricity market. All the coefficients of the cost and benefit functions are considered as random. A thorough analysis on the effects that one's position toward risk has on supply/demand levels, LMPs, and elasticities of the supply and demand curves is also made.

Finally, Chapter 5 presents a summary on the contributions of this work and gives directions for future research.

Appendix A gives some background on stochastic programming. Appendix B explores the possibility that generation and transmission expansion projects be considered as competitors rather than as interchangeable options. Appendix C presents the system's data for all the numerical examples presented in this work. In Appendix D, to further validate the models presented in this thesis, two large scale systems are solved; the one is a 57-node system and the other is a 118-node system.

# Chapter 2

# Centralized Power System Planning: A Deterministic Approach

Centralized planning in a vertically integrated industry plans for both generation and transmission expansion. The common practice is that after a generation plan is set, transmission expansion planning is carried out. In this sense, it can be said that generation and transmission expansion "go walking hand in hand" since, by the time the transmission planning is made, the location and timing for new generating units, and the closing of old ones, is perfectly known. A deterministic approach to centralized power system planning assumes that the future is either perfectly known, or it can be perfectly predicted. Hence, all the forecasted parameters, like future demand, are fixed known values.

For the generation expansion case, the system planner decides the optimal investment levels from various types of power plants; he has to choose from among different technologies. Therefore, generation expansion is motivated by quantity and relies on internalizing these quantity decisions [21, 22]. For the transmission expansion case, the system planner determines the timing, siting, and number of new circuits to be added. In both cases, the cost of the expansion is then passed on to the consumers who are charged a regulated tariff [23, 24].

Since the vertically integrated electricity industry plans for both generation and transmission, they can be considered as a complement of each other and in fact be combined into a single planning methodology. When generation expansion planning is combined with transmission expansion planning, the main objective is to decide whether to invest in new generation, new transmission, or a combination of both to ensure no load curtailment while minimizing the variable generating cost. In the following sections, this is formulated as a deterministic single-stage mixed-integer nonlinear programming problem.

This chapter is organized as follows. Section 2.1 explains the use of the Load Duration Curve (LDC) in single nodal point generation planning, and when the transmission network is considered. Capacity-based screening curves are discussed in order to explain how the fixed and variable costs of a generating unit are obtained. The annual equivalent value method is used to compute these costs. A simple static model is used to understand the rationale behind generation expansion planning. Afterwards, a multistage model is stated and fully explained. Finally, this multistage model is simplified into a single-stage model. In Section 2.2, a linearized version of the AC power flow, the DC power flow, is obtained. Using the DC power flow, a single-stage transmission expansion model is formulated. This model assumes a fixed capacity factor for the generating units, *i.e.*, only one operating mode. Section 2.3 combines the generation and transmission expansion problems into one single optimization model. The mixed-integer optimization model considers the various operating modes represented by the LDC and incorporates the transmission network using the DC model. Finally, Section 2.4 gives some concluding remarks.

## 2.1 Deterministic Generation Expansion

In this section, the basis are laid in order to formulate the deterministic generation expansion problem. Based on some of the most common models for generation expansion, a multistage generation expansion model is stated and fully explained. At the end of this section, the multistage model is simplified into a single-stage generation expansion model for the vertically integrated electricity industry.

### 2.1.1 The Load Duration Curve

The demand for electricity varies over time. LDCs are used to represent the operating conditions of a power system over time as long as the interest is only in the total capacity needed to meet future demands. LDCs are obtained from the hourly data of power demand over a period of time.

The LDC plots the number of hours (percentage of hours per year) that the load equals or exceeds a given level of demand (MW). An LDC can be thought as the probability of finding load above a certain level. It is customary to show on the vertical axis of this curve the load, and on the horizontal axis the duration either in hours or as a percentage of the hours per year. The amount of energy to be generated by each unit equals the area under the curve. A typical load duration curve is shown in Figure 2.1.

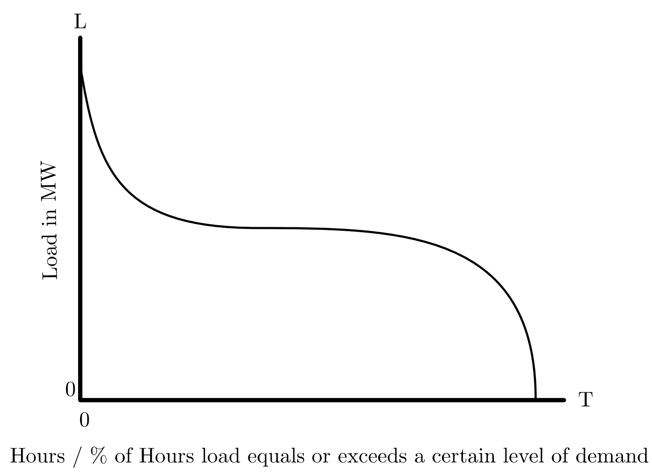


Figure 2.1: Typical load duration curve.

In [2], an example on how to construct a piecewise LDC is shown. A piecewise approximation of the LDC is shown in Figure 2.2.

The LDC is used in the short-run to determine the optimal dispatch of generation capacity. In the long-run, the LDC is used to decide the optimal level of investment in new generation, the type of power plant, and the number of each type [25]. The LDC can be used in generation expansion planning when all the load and all the generating units are assumed to be connected at the same node (single nodal point generation planning), or under the assumption that there is one LDC per major interconnecting node. In the case of the single nodal point generation planning, the geographical factor and the cost of transmission are ignored. The single nodal point generation planning is suitable when the power system is either strong or spans over a small region [26]. Load can be represented

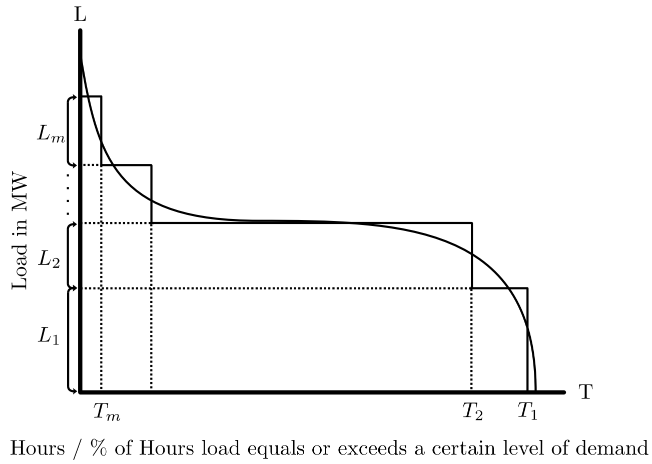


Figure 2.2: Piecewise approximation of the load duration curve.

over a year (8760 hrs), over the life cycle of the generating plants, or over the representation of two typical days per month (576 hrs) [4, 5]. By observing the LDC, it can be seen that load is made up of a constant component (load that is always present) and a fluctuating component (load that varies over time). The constant component is the base load demand while the fluctuating component is made up of the mid load demand and the peak demand. Note that, to satisfy the demand, some plants run all the time (base load plants), others run about half the time (mid load plants), while others only run for a short period of time during the high load demand (peaking plants). Peaking plants, like gas-fired combustion or internal combustion units, have low fixed costs and high operating costs; base load plants, like nuclear, hydroelectric, or coal fired units, have high fixed costs but very low operating costs; and mid load plants, like old coal-fired or oil-fired units, are somewhere in between. The capacity factor of a generating plant is determined by the load it serves. The capacity factor is defined as the percentage of hours of the year a generating plant serves a load [21, 22, 27].

### 2.1.2 Economic Assessment Methods

Economic assessment methods are used as a tool to select an optimal investment scheme when many technologies are available to choose from. There are three different types of economic assessment methods: i) the *static assessment method*, which ignores the time value of money; ii) the *dynamic assessment method*, which considers the time value of money; and iii) the *stochastic assessment method*, which considers the time value of money and uncertainty in some of the parameters. By using the probability distribution function of the uncertain variables, a probabilistic analysis approach can be used with the stochastic assessment method to valuate the profit of an investment.

When the time value of money is to be taken into consideration, one must bring all the cash flow in different time frames to the same time frame, *i.e.*, a payment in the future at the end of the year $n$ and a set of yearly instalments for $n$ years can be brought to the present in order to compare them.

The annual equivalent value method converts all the cost during the lifetime of a plant (or the duration of the planning period) into an annual cost and then compares all the different projects. The future cost $F$ at the end for year $n$ can be brought to the present $P$ using

$$P = F\frac{1}{(1+i)^n}, \tag{2.1}$$

where $i$ is the discount rate.

Once this is done, the annuities can be obtained using

$$A = P\frac{i(1+i)^n}{(1+i)^n - 1}, \tag{2.2}$$

where $A$ is the annual value [26].

### 2.1.3 Screening Curves

Screening curves can be used to identify the fixed and variable costs for a given generating unit as a function of the capacity factor. As mentioned before, the capacity factor is the percentage of the hours in the year that a generator serves a load, consequently, the capacity factor is determined by the load. A screening curve plots the average cost *vs.* the capacity factor. The two types of screening curves are: capacity-based and energy-based.

*Capacity-based screening curves* plot the average cost of using the *capacity* of the plant and are represented with the curve $AC_{\text{Capacity}} = FC + cf \times VC$. This type of screening curves can be used to determine optimal investment levels in different generating technologies based on the optimal durations a certain technology should serve each operating mode.

These curves can also be used to compare generation costs between different generating technologies. Figure 2.3 shows a typical capacity-based screening curve.

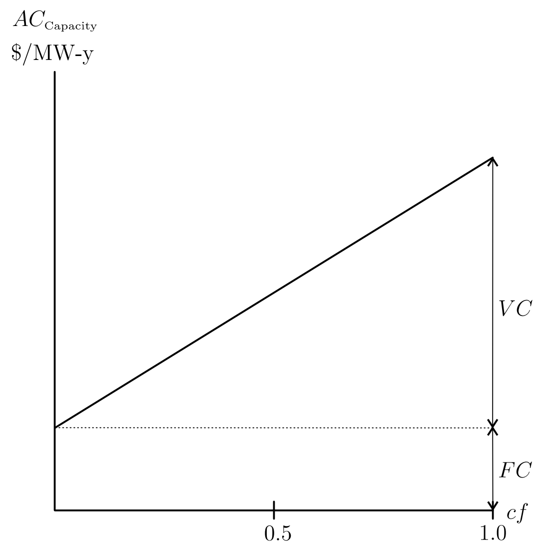


Figure 2.3: Capacity-based screening curve.

From Figure 2.3, it can be seen that the capacity-based screening curve is formed by a fixed component and a variable component. The fixed component is the investment cost. Usually, the investment cost is annualized either over the plant life or the planning period. The variable component is the variable operating cost whose biggest and most significative part is the fuel cost. Since investment plans range from ten to thirty years, the annualized fuel cost has to be levelized to take into account the effects of inflation. The uniform levelized annual equivalent of the fuel cost is

$$LAfc = fc\frac{1-\left[\frac{(1+a)}{1+i}\right]^n}{(i-a)}\frac{i(1+i)^n}{(1+i)^n-1}, \tag{2.3}$$

where $LAfc$ is the levelized annual fuel cost, $fc$ is the fuel cost in the first year, $a$ is the inflation rate, $i$ is the present worth rate, and $n$ is the number of years of the levelization. Equation (2.3) transforms an inflation series into annuities.

The following example shows how to obtain a capacity-based screening curve. Consider a coal-fired generating unit with an investment cost of capacity of 1500 \$/KW over a life plant of twenty years. Assume a fuel cost of 2 \$/MBtu, a heat rate of 9500 Btu/KWh, a present worth rate of 10%, and a fuel price escalation of 6%.

The fixed cost, computed from the investment cost of capacity using Equation (2.2), is

$$FC = 1500\left(\frac{0.1(1+0.1)^{20}}{(1+0.1)^{20}-1}\right) = 176.1894 \text{ \$/KW-y} = 0.1761894 \text{ M\$/MW-y}. \qquad (2.4)$$

During the first year, considering full time operation and with the fuel consumption rate given, the fuel cost is

$$\begin{aligned} fc &= (2)(9500)\left(\frac{\$}{\text{MBtu}}\right)\left(\frac{\text{Btu}}{\text{KWh}}\right)\left(\frac{1\text{ MBtu}}{1\times 10^{6}\text{ Btu}}\right)\left(\frac{1\times 10^{3}\text{ KWh}}{1\text{ MWh}}\right)\left(\frac{8760\text{ h}}{1\text{ y}}\right) \\ &= 166440 \text{ \$/MW-y} \\ &= 0.16644 \text{ M\$/MW-y}. \qquad (2.5) \end{aligned}$$

The uniform levelized annual fuel cost, using equation (2.3), is

$$\begin{aligned} VC = LAfc &= 0.16644\frac{1-\left[\frac{(1+0.06)}{1+0.1}\right]^{20}}{(0.1-0.06)}\frac{0.1(1+0.1)^{20}}{(1+0.1)^{20}-1} \\ &= (0.16644)(1.537) \\ &= 0.2558 \text{ M\$/MW-y}. \qquad (2.6) \end{aligned}$$

The capacity-based screening curve, from Equations (2.4) and (2.6), is

$$AC_{\text{Capacity}} = 0.1761894 + cf \times 0.2558 \text{ M\$/MW-y}. \qquad (2.7)$$

*Energy-based screening curves* show the average cost of the energy produced by a generating unit. They are represented by the curve $AC_{\text{Energy}} = \frac{FC}{cf} + VC$. Energy-based screening curves are very useful when assessing alternative technologies (nuclear, wind, solar or the like) in electricity markets. The capacity factors for alternative technologies are technical dependant rather than market dependant. This is because their variable costs are usually below the market price so they run whenever they are technically capable. To asses the economics of alternative technologies, the average cost of energy is compared with the average price of the market. Figure 2.4 shows a typical energy-based screening curve.

It is worth mentioning that when comparing several technologies using either the capacity-based or the energy-based screening curves, both curves intersect at the same point, that is, at the capacity factor at which one generating unit becomes more economical than the other. For a detailed discussion on screening curves please refer to [4, 22].

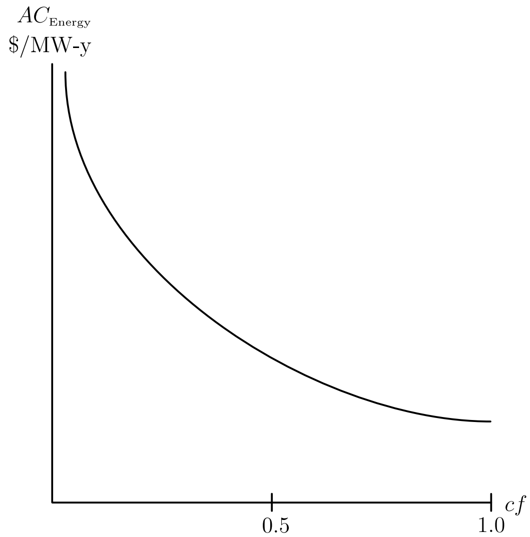


Figure 2.4: Energy-based screening curve.

### 2.1.4 Static Model

Having a static deterministic model implies that decisions are made only once and the future is assumed to be perfectly known. These type of models are the easiest to implement and the computational effort required to solve them is minimum. Deterministic models are helpful, though, to identify all the different aspects involved in generation expansion planning. Assuming a single nodal point generation planing, *i.e.*, omitting the transmission network, one can identify three basic parameters. These are: i) the investment cost, ii) the operating cost, and iii) the capacity factor. The generation expansion problem, using the LDC, is to find the type of power plant (coal, gas, oil, nuclear, hydro, renewable and so forth) that minimizes the total cost of producing 1 MW during the time $T$ of each operating mode (base, mid, and peak load). Assuming that there is no existing generation, this can be mathematically represented as

$$\min \quad \{r_{nt}x_{nt}n_{nt} + cf_m q_{nt} g_{m,nt}\}\,,\ \forall\ nt, m, \tag{2.8}$$

where $m \in \mathcal{M}$, $nt \in \mathcal{N}_t$, and $r_{nt}$ and $q_{nt}$ are obtained from capacity-based screening curves.

A logical result to (2.8), just by looking at the LDC, is to cover the base load demand (large values of $T_m$) with equipment that has low operating cost but hight investment cost, and the peak load demand (small values of $T_m$) with equipment that has low investment cost but high operating cost.

### 2.1.5 Multistage Model

When having in view long-term periods of investment, there are at least four reasons why the generation expansion problem may be addressed as a multistage program: i) long-term evolution of equipment costs, ii) long-term evolution of the load curve, iii) appearance of new technology, and iv) obsolescence of currently available equipment [5]. Based on the models in [5, 28], and ignoring the transmission network, the following multistage deterministic model is proposed.

The *objective function* to be minimized, Equation (2.9), is made up of the investment cost in new generating units and the generating cost of existing and newly installed generating capacity over the planning period

$$\min \sum_{t,nt} r_{nt}^t w_{nt}^t + \sum_{t,m} cf_m^t \left( \sum_{nt} q_{nt}^t g_{m,nt}^t + \sum_{et} q_{et}^t g_{m,et}^t \right). \tag{2.9}$$

The *installed capacity constraint*, Equation (2.10), describes the actual installed capacity of new technology that is available at each stage

$$w_{nt}^t = w_{nt}^{t-1} + x_{nt} n_{nt}^t, \ \forall \ nt, t. \tag{2.10}$$

The *peak demand constraint*, Equation (2.11), establishes that the capacity of the existing and newly installed generating units, affected by their equivalent availability, must be at least equal to the peak demand of every stage

$$\sum_{et} \alpha_{et} y_{et}^t + \sum_{nt} \alpha_{nt} w_{nt}^t \geq P^t, \ \forall \ t. \tag{2.11}$$

Equation (2.12) shows the *power balance constraint*. It establishes that the power generated by the existent and newly installed generating capacity equals the power demand of each operating mode for every stage

$$\sum_{nt} g_{m,nt}^t + \sum_{et} g_{m,et}^t = L_m^t, \ \forall \ m, t. \tag{2.12}$$

The *budget constraint*, Equation (2.13), places an upper limit in the fixed payments to be made each stage

$$\sum_{nt} r^t_{nt} w^t_{nt} \leq B^t, \ \forall \ t. \tag{2.13}$$

Equations (2.14) and (2.15) are the *derated capacity constraints*; they place an upper limit in the power generated at each stage by the existing and newly installed generating units that takes into account their derated capacity. The capacity of any generating unit is affected by its equivalent availability. Because of a component failure, or a similar condition, the output of an unit may be reduced. This type of outage results in the derating of the unit. The equivalent availability of a generating unit accounts for such derating outages [4]. It is worth mentioning that generating capacity is defined as the potential to deliver power. Consequently, it is measured in watts and there is no inconsistency in units

$$\sum_{m} (g^t_{m,nt}) - \alpha_{nt} w^t_{nt} \leq 0, \ \forall \ nt, t, \tag{2.14}$$

$$\sum_{m} (g^t_{m,et}) - \alpha_{et} y^t_{et} \leq 0, \ \forall \ et, t. \tag{2.15}$$

The relationship between indices and sets is as shown in Equation (2.16)

$$et \in \mathcal{E}_t; \ m \in \mathcal{M}; \ nt \in \mathcal{N}_t; \ t \in \mathcal{T}. \tag{2.16}$$

The fact that $y^t_{et}$ can be updated at every stage accounts for changes in the existing capacity, *i.e.*, the decommissioning of some units. If it is assumed that the planning period is less than the lifetime of the existing equipment, then $y^t_{et}$ remains unchanged.

It is worth mentioning that, in a stochastic model, there is a difference between *stages* and *periods*. In a two-stage stochastic model, the first stage decisions (investment decisions) are deterministic here-and-now decisions that do not change. The second stage decisions (operational decisions) are reactions to these fist stage decisions. However, each stage can be composed of several periods. For instance, if the whole planning period for the capacity expansion problem is made up of 36 months, the first stage is composed, say, of 6 months while the second stage is composed of 30 months. A Multistage model is needed if it is foreseen that the investment decisions need to be adjusted in the future.

### 2.1.6 Single-Stage Model

The following single-stage model is formulated as a benchmark in order to compare deterministic single-stage models with stochastic two-stage models.

*Objective function*

$$\min \sum_{nt} r_{nt} x_{nt} n_{nt} + \sum_{m} cf_m \left( \sum_{nt} q_{nt} g_{m,nt} + \sum_{et} q_{et} g_{m,et} \right). \tag{2.17}$$

Note that the parameters $q_{et}$, $q_{nt}$, and $r_{nt}$ in Equation 2.17 are obtained from capacity-based screening curves using the annual equivalent value method. Note also that in Equation (2.17), the annualized fixed cost of existing technology does not appear. This is because that is a fixed constant value dependant solely on the existing generating capacity, therefore it cannot be minimized.

*Peak demand constraint*

$$\sum_{et} \alpha_{et} y_{et} + \sum_{nt} \alpha_{nt} x_{nt} n_{nt} \geq P. \tag{2.18}$$

*Power balance constraint*

$$\sum_{nt} g_{m,nt} + \sum_{et} g_{m,et} = L_m, \ \forall \ m. \tag{2.19}$$

*Budget constraint*

$$\sum_{nt} r_{nt} x_{nt} n_{nt} \leq B. \tag{2.20}$$

*Derated capacity constraints*

$$\sum_{m} (g_{m,nt}) - \alpha_{nt} x_{nt} n_{nt} \leq 0, \ \forall \ nt, \tag{2.21}$$

$$\sum_{m} (g_{m,et}) - \alpha_{et} y_{et} \leq 0, \ \forall \ et. \tag{2.22}$$

The relationship between indices and sets is shown in Equation (2.23)

$$et \in \mathcal{E}_t; \ m \in \mathcal{M}; \ nt \in \mathcal{N}_t. \tag{2.23}$$

## 2.2 Deterministic Transmission Expansion

The goal of transmission expansion in a vertically integrated electricity industry is to minimize the operating cost of the existing generating units, and the investment cost in new transmission while meeting operational constraints. This section presents the basis for deterministic transmission expansion. A DC power flow model is used in order to formulate a single-stage deterministic optimization model.

### 2.2.1 AC Power Flow in a Transmission Line

Power is defined as the rate of flow of energy. Since power is a flow, it is measured in watts. Consider the $\pi$ equivalent circuit for long and medium transmission lines shown in Figure 2.5. The complex power, $S_{i\text{-}j}$, that flows in the transmission line that connects Node $i$ to Node $j$ is

$$S_{i\text{-}j} = P_{i\text{-}j} + jQ_{i\text{-}j}, \tag{2.24}$$

where $P_{i\text{-}j}$ is the active power and $Q_{i\text{-}j}$ is the reactive power.

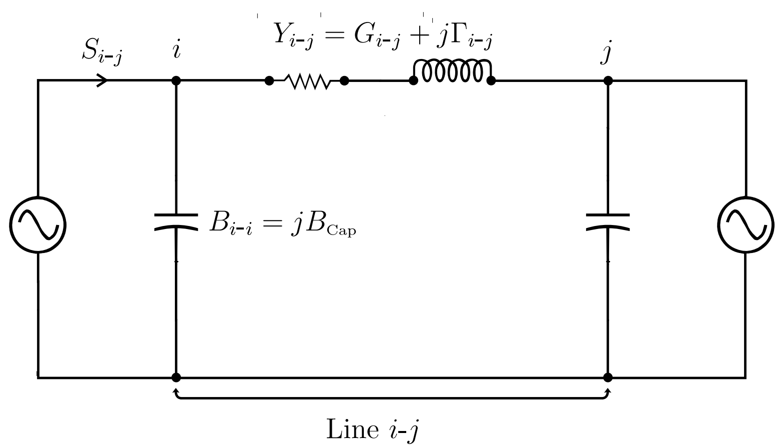


Figure 2.5: $\pi$ circuit for transmission lines.

The active power that flows from Node $i$ to Node $j$ is

$$P_{i\text{-}j} = G_{i\text{-}j}\left[|V_i|^2 - |V_i||V_j|\cos(\delta_i - \delta_j)\right] - \Gamma_{i\text{-}j}|V_i||V_j|\sin(\delta_i - \delta_j), \tag{2.25}$$

where $G_{i\text{-}j} = |Y_{i\text{-}j}| \cos\theta_{i\text{-}j}$ and $\Gamma_{i\text{-}j} = |Y_{i\text{-}j}| \sin\theta_{i\text{-}j}$ are the conductance and the susceptance of the transmission line $i$-$j$, respectively; $V_i = |V_i|\cos(\delta_i) + j|V_i|\sin(\delta_i)$ is the voltage at Node $i$; and $V_j = |V_j|\cos(\delta_j) + j|V_j|\sin(\delta_j)$ is the voltage at Node $j$.

The reactive power that flows from Node $i$ to Node $j$ is

$$Q_{i\text{-}j} = \Gamma_{i\text{-}j}\left[-|V_i|^2 + |V_i||V_j|\cos(\delta_i - \delta_j)\right] - G_{i\text{-}j}|V_i||V_j|\sin(\delta_i - \delta_j) - |V_i|^2|B_{\text{Cap}}|, \quad (2.26)$$

where $B_{\text{Cap}} = |B_{\text{Cap}}|\sin(\theta_{\text{Cap}})$, and $\theta_{\text{Cap}} = \frac{\pi}{2}$ [29].

The relationship between the admittance and the impedance is $Y_{i\text{-}j} = Z_{i\text{-}j}^{-1}$. From complex variable theory, and defining $Z_{i\text{-}j} = R_{i\text{-}j} + jX_{i\text{-}j}$, the conductance and the susceptance are

$$G_{i\text{-}j} = \frac{R_{i\text{-}j}}{R_{i\text{-}j}^2 + X_{i\text{-}j}^2}, \quad (2.27)$$

$$\Gamma_{i\text{-}j} = -\frac{X_{i\text{-}j}}{R_{i\text{-}j}^2 + X_{i\text{-}j}^2}. \quad (2.28)$$

A linearized version of the AC power flow—the DC power flow—dropping Equation (2.26) and keeping Equation (2.25), can be obtained under the assumption that the power angle $(\delta_i - \delta_j)$ is small in magnitude, and the voltage magnitudes at the sending and receiving nodes are 1 p.u. If $(\delta_i - \delta_j)$ is small one gets

$$\cos(\delta_i - \delta_j) \approx 1, \quad (2.29)$$

$$\sin(\delta_i - \delta_j) \approx \delta_i - \delta_j. \quad (2.30)$$

Under the previous assumptions, Equation (2.25) becomes

$$P_{i\text{-}j} = -\Gamma_{i\text{-}j}(\delta_i - \delta_j). \quad (2.31)$$

Combining Equations (2.28) and (2.31), and assuming that the resistance of the line is negligible, one gets the linearized version of the active power that flows from Node $i$ to Node $j$, that is

$$P_{i\text{-}j} = \frac{1}{X_{i\text{-}j}}(\delta_i - \delta_j). \quad (2.32)$$

### 2.2.2 Single-Stage Model

The single-stage deterministic model presented in this section is based on the initial work by Garver [30], who used linear programming to solve a transportation model for transmission expansion, and Villasana *et al.* [31], who used linear programming to solve a model which combined DC power flows with the transportation model. The model in [32] and [33], with some modifications, can be applied in a vertically integrated industry. It is assumed that the rights-of-way where new lines can be built are given. The proposed single-stage deterministic model is described next.

The *objective function*, Equation (2.33), minimizes the investment cost in new transmission lines and in the annualized variable production cost with an assumed fixed capacity factor for each existing generating unit

$$\min \quad \sum_{i\text{-}j} c_{i\text{-}j} n_{i\text{-}j} + \sum_{et,h} q_{et} g_{et,h}. \tag{2.33}$$

The *nodal balance constraint* is shown in Equation (2.34)

$$\boldsymbol{S}\boldsymbol{f} + \boldsymbol{g} = \boldsymbol{d}. \tag{2.34}$$

The *budget constraint*, shown in Equation (2.35), imposes an upper limit on the investment to be made each year in new transmission lines

$$\sum_{i\text{-}j} c_{i\text{-}j} n_{i\text{-}j} \leq B. \tag{2.35}$$

Equation (2.36) shows the *power flow constraint* for each transmission line; notice the nonlinearity due to the multiplication of integer variables with continuous variables

$$f_{i\text{-}j} - \Gamma_{i\text{-}j}(n^0_{i\text{-}j} + n_{i\text{-}j})(\delta_i - \delta_j) = 0, \ \forall \ i\text{-}j. \tag{2.36}$$

Equations (2.37) and (2.38) are the *transmission line capacity constraints*. The power loading of a transmission line depends, among other things, on thermal limits. Thermal limits are used to avoid damaging the conductor due to annealing and to avoid excessive sag [4]

$$f_{i\text{-}j} \leq (n^0_{i\text{-}j} + n_{i\text{-}j})\overline{f}_{i\text{-}j}, \ \forall \ i\text{-}j, \tag{2.37}$$

$$-f_{i\text{-}j} \leq (n^0_{i\text{-}j} + n_{i\text{-}j})\overline{f}_{i\text{-}j}, \ \forall \ i\text{-}j. \tag{2.38}$$

Stability limits are used to limit the power angle between sending and receiving nodes. In uncompensated lines, it is common practice to limit the power angle to a value no greater than 45° [34]. The *stability constraint* is shown in Equation (2.39)

$$-\frac{1}{4}\pi \leq (\delta_i - \delta_j) \leq \frac{1}{4}\pi, \ \forall \ i\text{-}j. \tag{2.39}$$

The *derated capacity constraint* is shown in Equation (2.40)

$$g_{et,h} - \alpha_{et} y_{et,h} \leq 0, \ \forall \ et, h. \tag{2.40}$$

*Upper and lower limit constraints* can be added to limit the values of some variables of the model such as the number of circuits to be added in each right-of-way, the capacity of each generator that is effectively used, and the nodal angles at each bus. These constraints are shown in Equations (2.41)–(2.43)

$$0 \leq n_{i\text{-}j} \leq \overline{n}_{i\text{-}j} \ \forall \ i\text{-}j, \tag{2.41}$$

$$g_{et,h} \geq 0 \ \forall \ et, h, \tag{2.42}$$

$$-2\pi \leq \delta_\nu \leq 2\pi, \ \forall \ \nu. \tag{2.43}$$

Equation (2.44) shows the relationship between indices and sets

$$et \in \mathcal{E}_t; \ h \in \mathcal{H}; \ h, i, j, \nu \in \mathcal{N}; \ i\text{-}j \in \mathcal{I}; \ \mathcal{H}, \mathcal{I} \subset \mathcal{N}. \tag{2.44}$$

## 2.3 Deterministic Generation and Transmission Expansion

As stated before, a vertically integrated industry plans for both generation and transmission expansion. Consequently, a joint model that plans for both at the same time can be implemented. This section combines the two models for generation and transmission expansion in a single-stage optimization model; the model assumes that there is one LDC per node and includes the transmission network using a DC model.

### 2.3.1 Single-Stage Model

If one wants to combine into one single optimization model the generation and transmission expansion problem, then one is interested in minimizing the investment cost in new transmission lines and in new generating units as well as in the generation cost of existing and newly installed generating capacity [35]. The following optimization model accounts for the different operating modes present in the LDC while incorporating the transmission network using a DC model.

*Objective function*

$$\min \quad \sum_{i\text{-}j} c_{i\text{-}j} n_{i\text{-}j} + \sum_{nt,p} r_{nt} x_{nt} n_{nt,p} + \sum_{m} cf_m \left( \sum_{et,h} q_{et} g_{m,et,h} + \sum_{nt,p} q_{nt} g_{m,nt,p} \right). \tag{2.45}$$

*Budget constraint*

$$\sum_{i\text{-}j} c_{i\text{-}j} n_{i\text{-}j} + \sum_{nt,p} r_{nt} x_{nt} n_{nt,p} \leq B. \tag{2.46}$$

Unlike before, the budget constraint now considers an upper limit in the investment to be made in new transmission lines and in new generating units.

*Derated capacity constraints*

$$\sum_{m} (g_{m,nt,p}) - \alpha_{nt} x_{nt} n_{nt,p} \leq 0, \ \forall \ nt, p, \tag{2.47}$$

$$\sum_{m} (g_{m,et,h}) - \alpha_{et} y_{et,h} \leq 0, \ \forall \ et, h. \tag{2.48}$$

*Nodal balance constraint*

$$\boldsymbol{S f_m} + \boldsymbol{g_m} = \boldsymbol{d_m}, \ \forall \ m. \tag{2.49}$$

*Power flow constraint*

$$f_{m,i\text{-}j} - \Gamma_{i\text{-}j}(n^0_{i\text{-}j} + n_{i\text{-}j})(\delta_{m,i} - \delta_{m,j}) = 0, \ \forall \ m, i\text{-}j. \tag{2.50}$$

Notice the nonlinearity due to the multiplication of integer variables with continuous variables.

*Transmission line capacity constraints*

$$f_{m,i\text{-}j} \leq (n^0_{i\text{-}j} + n_{i\text{-}j})\overline{f}_{i\text{-}j}, \ \forall \ m, i\text{-}j, \tag{2.51}$$

$$-f_{m,i\text{-}j} \leq (n^0_{i\text{-}j} + n_{i\text{-}j})\overline{f}_{i\text{-}j}, \ \forall \ m, i\text{-}j, \tag{2.52}$$

*Stability constraint*

$$-\frac{1}{4}\pi \leq (\delta_{m,i} - \delta_{m,j}) \leq \frac{1}{4}\pi,\ \forall\ m, i\text{-}j. \tag{2.53}$$

*Upper and lower limit constraints*

$$0 \leq n_{i\text{-}j} \leq \overline{n}_{i\text{-}j},\ \forall\ i\text{-}j, \tag{2.54}$$

$$g_{m,et,h} \geq 0,\ \forall\ m, et, h, \tag{2.55}$$

$$g_{m,nt,p} \geq 0,\ \forall\ m, nt, p, \tag{2.56}$$

$$-2\pi \leq \delta_{m,\nu} \leq 2\pi,\ \forall\ m, \nu. \tag{2.57}$$

Relationship between indices and sets

$$et \in \mathcal{E}_t;\ h \in \mathcal{H};\ h, i, j, p, \nu \in \mathcal{N};\ i\text{-}j \in \mathcal{I};\ m \in \mathcal{M};\ nt \in \mathcal{N}_t;\ p \in \mathcal{P};\ \mathcal{H}, \mathcal{I}, \mathcal{P} \subset \mathcal{N}. \tag{2.58}$$

## 2.4 Concluding Remarks

This chapter presents deterministic models for generation, transmission, and generation and transmission expansion. The goal is to identify all the modelling parameters in order to build more complex stochastic models and to have a benchmark to compare the stochastic models presented in Chapter 3. The main contribution of this chapter is the joint single-stage mixed-integer nonlinear program for generation and transmission expansion shown in Section 2.3.1. This model includes the transmission network using a DC model and assumes that there is one LDC per node. Hence, several operating modes (as many as there are in the linearized LDC) can be considered in order to choose the right mixture of generating technologies.

# Chapter 3

# Centralized Power System Planning: A Stochastic Approach

Planning for the real life is full of random events. The longer the period over which these events are forecasted, the greater the risk. One can shield against risk by implementing stochastic models, using probabilistic constraints, and applying risk minimization techniques. In centralized power system planning, the greatest source of uncertainty comes from the inability to accurately predict the demand levels. Stochastic models can be implemented in order to plan for all the foreseeable levels of demand. When planning for generation and transmission expansion, it is important to consider randomness in parameters such as the equivalent availability factors of generating units and the transmission capacity factors of transmission lines. These can be done by using probabilistic constraints. In order to minimize the risk, the mean-variance Markowitz theory can be implemented.

This chapter builds on the deterministic models presented in Chapter 2 and presents stochastic models for generation, transmission, and generation and transmission expansion for the vertically integrated industry. By means of numerical examples, the superiority of stochastic models with risk over deterministic models is stressed. Concepts such as the Expected Value of Perfect Information (EVPI) and the Value of the Stochastic Solution (VSS) are introduced as ways of quantifying uncertainty.

This chapter is organized as follows. Section 3.1 introduces the general concept of the mean-variance Markowitz theory. In Section 3.2, the deterministic equivalent of a probabilistic constraint for the particular case of a normally distributed random variable is obtained. Section 3.3 introduces the concepts of the VSS and the EVPI as a way of quantifying the importance of randomness. In Section 3.4, a multistage stochastic model for generation expansion is first introduced. Afterwards, using the block separability property of the multistage model, a two-stage stochastic model is obtained. Using the mean-variance Markowitz theory and probabilistic constraints, a two stage stochastic model that minimizes risk and accounts for the randomness of the equivalent availability factors is proposed. Some numerical examples are given. In Section 3.5, the transmission expansion problem is stated as a two-stage stochastic program. Like in Section 3.4, a risk parameter is introduced in the objective function and probabilistic constraints are included to account for the random equivalent availabilities and the random transmission capacity factors. The concepts of the VSS and the EVPI are illustrated by solving a six-node system. In Section 3.6, the two-stage stochastic programs for generation and transmission expansion are merged into a single optimization problem. The most complete model is a mixed-integer nonlinear program that includes risk minimization and probabilistic constraints. In this section, a six- and a 21-node systems[1] are solved and the VSS and EVPI are also obtained. Finally, Section 3.7 gives some concluding remarks.

## 3.1 Mean-Variance Markowitz Theory

Harry M. Markowitz developed the minimum variance portfolios theory in which the objective is to minimize the variance of the rate of return for a fixed expected rate of return. This approach makes the trade-offs between the mean and the variance explicit [36] – [38]. The variance of any random variable gives information about how far, or how wrong [1], that variable is from its mean. Large variance values imply that the random variable most likely will be far off from its mean incurring then in greater risk. Hence, the minimum variance approach can be viewed as a way of minimizing the risk in an investment project. Minimizing the risk (minimizing the variance) implies higher cost. In order to account for risk in a two-stage stochastic model,[2] a risk factor along with the variance are incorporated

[1]A 57- and a 118-node system are presented in Appendix D.

[2]For a detailed discussion on stochastic models, please refer to Appendix A.

into the objective function as follows [39]:

$$\min \; f(x) + E\{f(y)\} + \theta_r \sigma^2_{f(y)}, \tag{3.1}$$

where $f(x)$ is the first-stage decision (deterministic part), $E\{f(y)\}$ is the expected value of the second-stage decision (stochastic part), and $\sigma^2_{f(y)}$ is the variance of the second-stage decision. The parameter $\theta_r$ is a *risk* parameter and is chosen by the decision maker; it weighs the importance that the minimization of the variance has in the objective function. The higher the value of $\theta_r$, the more important the minimization of the variance.

The variance term in Equation (3.1) can be replaced by the standard deviation as long as $\theta_r$ is properly scaled.

## 3.2 Probabilistic Constraints

Some constraints in a stochastic program may be relaxed in the sense that they may not need to hold *surely*; instead, they may need to hold *almost surely*, say, with a reliability level. These constraints are known as probabilistic constraints. The general formulation for a single chance constraint is

$$\Pr\{A(\tilde{\xi})x \geq h(\tilde{\xi})\} \geq \alpha, \tag{3.2}$$

where $0 < \alpha < 1$ is the minimum acceptable probability for meeting the constraint.

To incorporate probabilistic constraints into a recourse problem, *i.e.*, the two-stage stochastic problem with fixed recourse, they first need to be transformed into their deterministic equivalents. The deterministic equivalent for the specific case of a single probabilistic constraint when $A(\tilde{\xi})$ is a standard normally distributed random variable and $h(\tilde{\xi}) = 0$ is

$$\bar{A}x - \Phi^{-1}(\alpha)\sqrt{x^T C x} \geq 0, \tag{3.3}$$

where $\bar{A}$ is the mean value of $A(\tilde{\xi})$, $C$ is the covariance matrix, and $\Phi^{-1}(\alpha)$ is the $\alpha$-quantile[3] of $A(\tilde{\xi})$. Please note that Equation (3.3) is only valid under these assumptions [5].

[3] For a random variable $\xi$, $\eta$ is its $\alpha$-quantile if and only if $\eta = \min\{x \mid F(x) \geq \alpha\}$ where $0 < \alpha < 1$ and $F(x)$ is the cumulative distribution function.

## 3.3 Quantifying the Importance of Randomness

There are two values of interest when solving stochastic problems that can quantify the importance of randomness. One is the *value of the stochastic solution* (VSS) and the other is the *expected value of perfect information* (EVPI).

The VSS is defined as *the difference between the expected result of using the Expected Value Solution* (EEV) *and the objective function value of the stochastic problem considering all the possible scenarios* (Stochastic Solution, SS, also referred as the here-and-now solution). The EEV is obtained from solving the stochastic problem, but considering the aggregate-level decisions fixed to the results obtained by the deterministic model with the average demand (expected values) [5, 40], that is

$$\mathrm{VSS} = \mathrm{EEV} - \mathrm{SS}. \tag{3.4}$$

The VSS is the cost of ignoring uncertainty in choosing a decision [5]; in other words, it computes the benefit of knowing the distributions of the stochastic variables [40].

The EVPI represents the loss of profit due to the presence of uncertainty or, in other words, measures the maximum amount a decision maker would be ready to pay in return for complete and accurate information about the future [5]. The EVPI measures the value of knowing the future with certainty [40]. The EVPI is the difference between the stochastic solution (SS) and the Wait and See Solution (WSS). The WSS is the mean value of all the deterministic solutions for each one of the expected values, that is

$$\mathrm{EVPI} = \mathrm{SS} - \mathrm{WSS}. \tag{3.5}$$

## 3.4 Stochastic Generation Expansion

The random long-term evolution of equipment cost, the long term uncertainty in the evolution of the LDC, the uncertainty on the appearance of new technologies, and the obsolescence of the existent equipment are enough reasons to justify a multistage stochastic program to model the generation expansion problem. In the model shown in Section 2.1.5, a number of variables can be considered as random. To give an example: the evolution of equipment cost introduces randomness in the parameter $r^t_{nt}$; evolution of fuel cost as well as other costs related with the production of energy also make the parameters $q^t_{nt}$ and $q^t_{et}$ random; even the equivalent availabilities, $\alpha_{nt}$ and $\alpha_{et}$, can be considered random since some plants may not deliver their nominal output due to a derated outage which is totally unpredictable. Another highly random parameter is the load $L^t_m$ for each operating mode. This leads to the formulation of the problem as a multistage stochastic model.

### 3.4.1 Multistage Model

Without any loss of generality, assume that only the levels of demand $\boldsymbol{L}^t_m$ are random[4] and that the probabilities of their occurrences as well as their expected values are known. Under these assumptions, the total capacity of new technology available at each stage $\boldsymbol{w}^t_{nt}$, the capacity of new and existing technology effectively used $\boldsymbol{g}^t_{m,nt}$ and $\boldsymbol{g}^t_{m,et}$, and the new generating capacity added $x_{nt}\boldsymbol{n}^t_{nt}$, become random variables dependent on the random vector $\boldsymbol{\xi}^t$ whose elements are the different levels of demand $\boldsymbol{L}^t_m$. Taking the expectation with respect to the random vector $\boldsymbol{\xi} = (\boldsymbol{\xi}^2, \ldots, \boldsymbol{\xi}^{|\mathcal{T}|})$, where the elements forming $\boldsymbol{\xi}^t$ are the demands $(\boldsymbol{L}^t_1, \ldots, \boldsymbol{L}^t_{|\mathcal{M}|})$, the multistage stochastic model can be written as

*Objective function*

$$\min \quad E_{\boldsymbol{\xi}} \left\{ \sum_{t,nt} r^t_{nt} \boldsymbol{w}^t_{nt} + \sum_{t,m} cf^t_m \left( \sum_{nt} q^t_{nt} \boldsymbol{g}^t_{m,nt} + \sum_{et} q^t_{et} \boldsymbol{g}^t_{m,et} \right) \right\}. \tag{3.6}$$

*Installed capacity constraint*

$$\boldsymbol{w}^t_{nt} = \boldsymbol{w}^{t-1}_{nt} + x_{nt} \boldsymbol{n}^t_{nt}, \ \forall \ nt, t. \tag{3.7}$$

*Peak demand constraint*

$$\sum_{et} \alpha_{et} y^t_{et} + \sum_{nt} \alpha_{nt} \boldsymbol{w}^t_{nt} \geq P^t, \ \forall \ t. \tag{3.8}$$

*Power balance constraint*

$$\sum_{nt} \boldsymbol{g}^t_{m,nt} + \sum_{et} \boldsymbol{g}^t_{m,et} = \boldsymbol{L}^t_m, \ \forall \ m, t. \tag{3.9}$$

[4]Only in this section and the following, bold font notation is used to indicate dependance on the random outcome $\omega$, *i.e.*, below average, average, and above average level of demand.

*Budget constraint*

$$\sum_{nt} r_{nt}^t \boldsymbol{w}_{nt}^t \leq B^t, \ \forall \ t. \tag{3.10}$$

*Derated capacity constraints*

$$\sum_m (\boldsymbol{g}_{m,nt}^t) - \alpha_{nt} \boldsymbol{w}_{nt}^t \leq 0, \ \forall \ nt, t, \tag{3.11}$$

$$\sum_m (\boldsymbol{g}_{m,et}^t) - \alpha_{et} y_{et}^t \leq 0, \ \forall \ et, t. \tag{3.12}$$

*Indices and sets*

$$et \in \mathcal{E}_t; \ m \in \mathcal{M}; \ nt \in \mathcal{N}_t; \ t \in \mathcal{T}. \tag{3.13}$$

However, making some simplifications and assumptions, the generation expansion problem can be transformed into a two-stage stochastic problem.

### 3.4.2 Two-Stage Model

The problem in (3.6)–(3.13) has a property known as *block separable recourse*. This property makes possible to transform a multistage stochastic program with fixed recourse into a two-stage stochastic program with fixed recourse. A multistage stochastic program has block separable recourse when the decision vectors can be split into aggregate-level decisions and detailed-level decisions.[5] The block separable recourse property implies that the detailed-level variables have no direct effect on future constraints or, in other words, it implies that the linkage between consecutive stages is weak. The aggregate-level variables can be grouped together and sent into the first stage. The first stage is then composed of the aggregate-level decisions while the second stage is composed of the detailed-level decisions. Because of this separation, a multistage stochastic problem can be transformed into a two-stage stochastic problem.

In the generation expansion problem, it is assumed that future demands do not depend on the past, *i.e.*, having high levels of demand during several previous consecutive stages does not necessarily imply that the demand in the following stages will be high too. Capacity carried over from stage $t-1$ to stage $t$ is not affected by the demand in stage $t$. Therefore, the decision to install new generating capacity, $x_{nt}\boldsymbol{n}_{nt}^t$, in the future does not

[5]Please refer to Appendix A for a detailed discussion.

depend on the outcomes up to stage $t$. Decisions on the amount of capacity to be installed can be made at the beginning of the planning period; consequently, the future only involves reactions to these decisions. The same $x_{nt}\boldsymbol{n}_{nt}^t$ must be optimal for any realization of $\boldsymbol{\xi}$. The only remaining stochastic decisions, $\boldsymbol{g}_{m,et}^t$ and $\boldsymbol{g}_{m,nt}^t$, are in the operation-level; these stochastic decisions now depend solely on each stage's capacity. The aggregate-level decisions, $x_{nt}\boldsymbol{n}_{nt}^t$ and $\boldsymbol{w}_{nt}^t$, can be pulled together into the first stage while the detailed-level decisions, $\boldsymbol{g}_{m,et}^t$ and $\boldsymbol{g}_{m,nt}^t$, can be pulled together into the second stage [5].

The objective function[6] of the two-stage stochastic model with fixed recourse is obtained by carrying the aggregate-level decisions into the first stage and taking the expectation over the detailed-level decisions. Define $z$ as

$$z = \sum_m cf_m \left( \sum_{nt} q_{nt} g_{m,nt} + \sum_{et} q_{et} g_{m,et} \right). \tag{3.14}$$

The expectation over the detailed-level decisions is [41]

$$\begin{aligned} E\{z\} &= \sum_k pr_k z_k \\ &= \sum_k pr_k \left( \sum_m cf_m \left( \sum_{nt} q_{nt} g_{m,nt} + \sum_{et} q_{et} g_{m,et} \right) \right) \\ &= \sum_{k,m} pr_k cf_m \left( \sum_{nt} q_{nt} g_{k,m,nt} + \sum_{et} q_{et} g_{k,m,et} \right), \end{aligned} \tag{3.15}$$

where $pr_k$ is the probability of occurrence of scenario[7] $k$, $\mathcal{K}$ is the set of all foreseeable scenarios, and $k \in \mathcal{K}$.

The two-stage stochastic program with fixed recourse is formally stated in Equations (3.16)–(3.22); it ignores the transmission network, *i.e.*, single nodal point generation planning. This model considers randomness in the levels of demand (right hand side) through the consideration of a number of foreseeable scenarios. The variables no longer depend on $\mathcal{T}$; the random variables are now dependant on $\mathcal{K}$.

[6]From now onward, the bold font notation is dropped in order to avoid cumbersome notation.

[7]It is assumed that a scenario is a possible occurrence that is given; this work is not concerned with the technique used to generate these scenarios.

*Objective function*

$$\sum_{nt} r_{nt} x_{nt} n_{nt} + \sum_{k,m} pr_k cf_m \left( \sum_{nt} q_{nt} g_{k,m,nt} + \sum_{et} q_{et} g_{k,m,et} \right). \quad (3.16)$$

*Peak demand constraint*

$$\sum_{et} \alpha_{et} y_{et} + \sum_{nt} \alpha_{nt} x_{nt} n_{nt} \geq P. \quad (3.17)$$

*Power balance constraint*

$$\sum_{nt} g_{k,m,nt} + \sum_{et} g_{k,m,et} = L_{k,m}, \ \forall \ k, m. \quad (3.18)$$

*Budget constraint*

$$\sum_{nt} r_{nt} x_{nt} n_{nt} \leq B. \quad (3.19)$$

*Derated capacity constraints*

$$\sum_{m} (g_{k,m,nt}) - \alpha_{nt} x_{nt} n_{nt} \leq 0, \ \forall \ k, nt, \quad (3.20)$$

$$\sum_{m} (g_{k,m,et}) - \alpha_{et} y_{et} \leq 0, \ \forall \ k, et. \quad (3.21)$$

*Indices and sets*

$$et \in \mathcal{E}_t; \ m \in \mathcal{M}; \ nt \in \mathcal{N}_t; \ k \in \mathcal{K}. \quad (3.22)$$

### 3.4.3 Deterministic Model *vs.* Two-Stage Stochastic Model

This section presents a numerical example that highlights the superiority of a two-stage stochastic model over a single-stage deterministic model for generation expansion in a vertically integrated industry.

The following assumptions are made: The transmission network is ignored. There are two existing generating units and there are three different technologies to choose from. The planning period is shorter than the life time of either the existing or new plants. The LDC is obtained over a typical year, that is, over 8760 hours. Three operating modes are considered: base, mid, and peak load demand. Each operating mode has three possible foreseeable scenarios: below, average, and above average demand. The expected values of the equivalent availabilities, $\alpha_{et}$ and $\alpha_{nt}$, are used. The budget constraint is not binding.

All the pertinent data is given in Appendix C.

The decision maker has to guess what the future demand would be. If a conservative approach is used, the decision maker invests for the average levels of demand. If a pessimistic approach is used, the decision maker invests for the above average levels of demand. Of course, a pessimistic approach implies higher investment cost but it satisfies the demands for the three different scenarios, although not in an economical way. If the decision maker wishes to save as much as possible in the investment, then he invests for the below average levels of demand. To show how cost varies depending on the levels of demand used in the investment plan, the single-stage deterministic generation expansion problem, shown in Equations (2.17)–(2.23), is solved[8] for each one of the three different foreseeable scenarios. The results are shown in Tables 3.1–3.3. For the below average level of demand, one unit of technology two is built. For the average level of demand, one unit of technology one is built. For the above average level of demand, one unit of technology one and one unit of technology two are built.

Table 3.1: Deterministic gen. exp., below average.

| Operating Mode† | *et* | | *nt* |
|---|---|---|---|
| | 1 | 3 | 2 |
| B‡ | 462.5 | 0.0 | 17.5 |
| M⋆ | 0.0 | 101.0 | 79.0 |
| P∗ | 0.0 | 80.0 | 0.0 |
| Overall Cost: $155.877 M. | | | |

† in MW; ‡ Base, ⋆ Mid, and ∗ Peak.

An alternative to solve this problem, is to take into account the probabilities of occurrence of the three possible scenarios by solving the two-stage stochastic model with fixed

[8]The models in this section are implemented in the optimization software GAMS using the MINOS solver [42].

Table 3.2: Deterministic gen. exp., average.

| Operating Mode† | *et* | | *nt* |
|---|---|---|---|
| | 1 | 3 | 1 |
| B‡ | 222.5 | 0.0 | 317.5 |
| M⋆ | 240.0 | 0.0 | 0.0 |
| P∗ | 0.0 | 0.0 | 140.0 |
| Overall Cost: $206.4916 M. | | | |

† in MW; ‡ Base, ⋆ Mid, and ∗ Peak.

Table 3.3: Deterministic gen. exp., above average.

| Operating Mode† | *et* | | *nt* | |
|---|---|---|---|---|
| | 1 | 3 | 1 | 2 |
| B‡ | 162.5 | 0.0 | 437.5 | 0.0 |
| M⋆ | 300.0 | 0.0 | 0.0 | 0.0 |
| P∗ | 0.0 | 78.5 | 25.0 | 96.5 |
| Overall Cost: $246.169 M. | | | | |

† in MW; ‡ Base, ⋆ Mid, and ∗ Peak.

recourse shown in Equations (3.16)–(3.22). The solution satisfies the three possible levels of demand and, hopefully, is cheaper than the pessimistic approach. The solution of the stochastic model is shown in Table 3.4. The stochastic solution is to build one unit of technology one and two units of technology two.

From the results it can be seen that the new capacity installed satisfies the three levels of demand and, comparing the overall costs,[9] $20.655 M are saved with respect to the deterministic solution for the above average level of demand. The results also suggest the decommissioning of the existing 200 MW of technology three.

[9]The overall cost equals the investment cost plus the expected variable generating cost.

Table 3.4: Stochastic generation expansion.

| Operating Mode† | *et* | | *nt* | |
|---|---|---|---|---|
| Below Average | 1 | 3 | 1 | 2 |
| B‡ | 382.5 | 0.0 | 97.5 | 0.0 |
| M⋆ | 0.0 | 0.0 | 180.0 | 0.0 |
| P* | 80.0 | 0.0 | 0.0 | 0.0 |
| Average | | | | |
| B‡ | 462.5 | 0.0 | 77.5 | 0.0 |
| M⋆ | 0.0 | 0.0 | 240.0 | 0.0 |
| P* | 0.0 | 0.0 | 140.0 | 0.0 |
| Above Average | | | | |
| B‡ | 462.5 | 0.0 | 137.5 | 0.0 |
| M⋆ | 0.0 | 0.0 | 300.0 | 0.0 |
| P* | 0.0 | 0.0 | 25.0 | 175.0 |
| Overall Cost: $225.514 M. | | | | |

† in MW; ‡ Base, ⋆ Mid, and ∗ Peak.

### 3.4.4 Two-Stage Model with Risk

In the generation expansion problem where risk minimization is introduced by means of the mean-variance Markowitz theory, the interest is in the minimization of the investment cost in new generation, the expected annualized production cost, and the variance of the annualized production cost. An investor that chooses a solution with low variance but high annualized production cost, is said to be *risk averse* (large values of $\theta_r$). An investor that chooses a solution with high variance but low annualized production cost, is said to be *risk preferring* (small values of $\theta_r$) [43].

The variance of $z$, as defined in Equation (3.14), is [6]

$$\begin{aligned}\sigma_z^2 &= E\{z^2\} - E^2\{z\} \\ &= \sum_k pr_k \left( \sum_m cf_m \left( \sum_{nt} q_{nt} g_{k,m,nt} + \sum_{et} q_{et} g_{k,m,et} \right) \right)^2 \\ &\quad - \left( \sum_{k,m} pr_k cf_m \left( \sum_{nt} q_{nt} g_{k,m,nt} + \sum_{et} q_{et} g_{k,m,et} \right) \right)^2 . \end{aligned} \tag{3.23}$$

From Equations (3.1), (3.15), and (3.23), the objective function of the two-stage stochastic model incorporating the variance and the risk parameter is shown in Equation (3.24); the constraints remain as in (3.17)–(3.22).

$$\begin{aligned}\min \quad & \sum_{nt} r_{nt} x_{nt} n_{nt} + \sum_{k,m} pr_k cf_m \left( \sum_{nt} q_{nt} g_{k,m,nt} + \sum_{et} q_{et} g_{k,m,et} \right) \\ & + \theta_r \left\{ \sum_k pr_k \left( \sum_m cf_m \left( \sum_{nt} q_{nt} g_{k,m,nt} + \sum_{et} q_{et} g_{k,m,et} \right) \right)^2 \right. \\ & \left. - \left( \sum_{k,m} pr_k cf_m \left( \sum_{nt} q_{nt} g_{k,m,nt} + \sum_{et} q_{et} g_{k,m,et} \right) \right)^2 \right\} . \end{aligned} \tag{3.24}$$

It can be shown that, by varying the risk parameter $\theta_r$, a wide range of solutions can be obtained. The value of the risk parameter is to be chosen by the decision maker. The risk parameter $\theta_r$ weighs the importance that the minimization of the variance as a risk measure has. The larger the value of $\theta_r$, the smaller the risk. Smaller risk implies higher cost. In Table 3.5, some solutions are generated for different values of $\theta_r$.

Table 3.5: Annualized generating cost, $\theta_r$ varies, gen. exp.

| Risk Parameter | Mean† | Std. Deviation† | Overall Cost‡ |
|---|---|---|---|
| $\theta_r = 0.00$ | 125.045 | 21.735 | 211.895 |
| $\theta_r = 0.03$ | 128.134 | 18.633 | 214.984 |
| $\theta_r = 0.04$ | 129.920 | 17.79 | 216.770 |
| $\theta_r = 0.05$ | 132.791 | 15.275 | 219.641 |
| $\theta_r = 0.06$ | 136.679 | 12.729 | 223.529 |
| $\theta_r = 0.10$ | 144.457 | 7.637 | 231.307 |

† in \$M-y; ‡ Investment cost plus annualized generating cost, in \$M-y.

From the results one can observe that as the risk aversion increases, the overall cost increases and the standard deviation decreases.[10]

[10]The square root of the variance is the standard deviation. Therefore, $\sqrt{\sigma_z^2} = \sigma_z$ is the standard deviation.

### 3.4.5 Two-Stage Model with Risk and Probabilistic Constraints

To consider randomness in the equivalent availability of the generating plants, one can incorporate probabilistic (chance) constraints to the two-stage stochastic model. The derated capacity constraints, Equations (3.20) and (3.21), can be modified so they hold almost surely with a specific reliability level. Assuming that the equivalent availability factor is a standard normally distributed random variable, the *probabilistic derated capacity constraints* are

$$\sum_m (g_{k,m,nt}) - \alpha_{nt} x_{nt} n_{nt} + \beta_g \left\{ \sigma^2_{\alpha_{nt}} (x_{nt} n_{nt})^2 \right\}^{\frac{1}{2}} \leq 0, \ \forall \ k, \ nt, \tag{3.25}$$

$$\sum_m (g_{k,m,et}) - \alpha_{et} y_{et} + \beta_g \left\{ \sigma^2_{\alpha_{et}} (y_{et})^2 \right\}^{\frac{1}{2}} \leq 0, \ \forall \ k, \ et. \tag{3.26}$$

The parameter $\beta_g$ is the $\alpha$-quantile of the standard normally distributed random variables $\alpha_{et}$ and $\alpha_{nt}$, assuming they are of the same technology. The value of $\beta_g$ depends on the minimum desired probability of satisfying the random (chance) constraints, *i.e.*, if $\beta_g$ equals 1.75, the probabilistic constraints are met with at least a 96% probability. As in the case of $\theta_r$, the decision maker is responsible for choosing the value $\beta_g$.

Note that, if the variances of $\alpha_{nt}$ and $\alpha_{et}$ are known, Equations (3.25) and (3.26) can be linearized. Hence, the linearized constraints are

$$\sum_m (g_{k,m,nt}) - \alpha_{nt} x_{nt} n_{nt} + \beta_g \sigma_{\alpha_{nt}} x_{nt} n_{nt} \leq 0, \ \forall \ k, nt, \tag{3.27}$$

$$\sum_m (g_{k,m,et}) - \alpha_{et} y_{et} + \beta_g \sigma_{\alpha_{et}} y_{et} \leq 0, \ \forall \ k, et. \tag{3.28}$$

In Table 3.6, some solutions for the model represented by Equations (3.24), (3.17)–(3.19), (3.22), (3.27), and (3.28) are shown. The parameter $\theta_r$ is fixed to a value of 0.05 while $\beta_g$ is allowed to vary. It can be seen that as the probability of meeting the chance constraints increases, the overall cost also increases.

For different values of $\theta_r$, and keeping $\beta_g$ fixed to a value of 1.3 (at least a 90% probability the chance constraints are met), the mean and the standard deviation of the annualized generating cost are obtained. The values are presented in Table 3.7. It can be observed that a risk averse attitude implies higher investment and operational costs.

Table 3.6: Annualized generating cost, $\beta_g$ varies, gen. exp.

| Probability | Mean† | Std. Deviation † | Overall Cost‡ |
|---|---|---|---|
| $\approx 76\%; \beta_g = 0.7$ | 146.652 | 15.275 | 249.822 |
| $\approx 84\%; \beta_g = 1.0$ | 153.248 | 15.275 | 264.578 |
| $\approx 90\%; \beta_g = 1.3$ | 160.200 | 15.275 | 279.690 |
| $\approx 96\%; \beta_g = 1.74$ | 175.078 | 15.275 | 302.728 |

† in \$M-y; ‡ Investment cost plus annualized generating cost, in \$M-y.

Table 3.7: Annualized generating cost, $\theta_r$ varies, gen. exp.

| Risk Parameter | Mean† | Standard Deviation† | Overall Cost‡ |
|---|---|---|---|
| $\theta_r = 0.00$ | 144.704 | 32.043 | 260.084 |
| $\theta_r = 0.03$ | 154.502 | 16.428 | 269.882 |
| $\theta_r = 0.04$ | 154.366 | 19.094 | 273.856 |
| $\theta_r = 0.05$ | 160.200 | 15.275 | 279.690 |
| $\theta_r = 0.06$ | 164.089 | 12.729 | 283.529 |
| $\theta_r = 0.10$ | 171.866 | 7.637 | 291.356 |

† in \$M-y; ‡ Investment cost plus annualized generating cost, in \$M-y.

## 3.5 Stochastic Transmission Expansion

When planning an expansion in a transmission network, a decision maker faces the same dilemma as when expanding generation. The level of demand is highly uncertain; to provide a safe operation of the power system, the decision maker might adopt a pessimistic approach and expand for the highest level of demand expected to happen, incurring then in high investment costs. This situation calls for the implementation of stochastic models.

In the following sections, like it is done in the case of generation expansion, the transmission expansion problem is stated first as a two-stage stochastic program, then it is extended to incorporate risk in the objective function, and finally probabilistic constraints are added to consider randomness in the transmission capacity factors of the transmission lines.

### 3.5.1 Two-Stage Model

In this section, a two-stage stochastic model with fixed recourse is formulated for the transmission expansion problem. This stochastic model considers uncertainty in the demand side; it has as first-stage decisions the investment in new transmission, and as second-stage decisions the annual estimate of generation in order to minimize the annualized variable production cost[11] of the existing generating plants based on the transmission capacity added. The second stage objective can be thought as the minimization of the expected annual production cost. Redefine $z$ as

$$z = \sum_{et,h} q_{et} g_{et,h}. \tag{3.29}$$

The expectation of $z$ is

$$E\left\{z\right\} = \sum_{k,et,h} pr_k q_{et} g_{k,et,h}. \tag{3.30}$$

The two-stage stochastic model for transmission expansion in a vertically integrated industry is formally stated next.

[11] A fixed capacity factor for each generating plant is assumed.

*Objective function*

$$\min \quad \sum_{i\text{-}j} c_{i\text{-}j} n_{i\text{-}j} + \sum_{k,et,h} pr_k q_{et} g_{k,et,h}. \tag{3.31}$$

*Nodal balance constraint*

$$\boldsymbol{S f}_k + \boldsymbol{g}_k = \boldsymbol{d}_k, \ \forall \ k. \tag{3.32}$$

*Budget constraint*

$$\sum_{i\text{-}j} c_{i\text{-}j} n_{i\text{-}j} \leq B. \tag{3.33}$$

*Power flow constraint*

$$f_{k,i\text{-}j} - \Gamma_{i\text{-}j}(n^0_{i\text{-}j} + n_{i\text{-}j})(\delta_{k,i} - \delta_{k,j}) = 0, \ \forall \ k, i\text{-}j. \tag{3.34}$$

*Transmission line capacity constraints*

$$f_{k,i\text{-}j} \leq (n^0_{i\text{-}j} + n_{i\text{-}j})\overline{f}_{i\text{-}j}, \ \forall \ k, i\text{-}j, \tag{3.35}$$

$$-f_{k,i\text{-}j} \leq (n^0_{i\text{-}j} + n_{i\text{-}j})\overline{f}_{i\text{-}j}, \ \forall \ k, i\text{-}j. \tag{3.36}$$

*Stability constraint*

$$-\frac{1}{4}\pi \leq (\delta_{k,i} - \delta_{k,j}) \leq \frac{1}{4}\pi, \ \forall \ k, i\text{-}j. \tag{3.37}$$

*Derated capacity constraint*

$$g_{k,et,h} - \alpha_{et} y_{et,h} \leq 0, \ \forall \ k, et, h. \tag{3.38}$$

*Upper and lower limit constraints*

$$g_{k,et,h} \geq 0, \ \forall \ k, et, h, \tag{3.39}$$

$$0 \leq n_{i\text{-}j} \leq \overline{n}_{i\text{-}j}, \ \forall \ i\text{-}j, \tag{3.40}$$

$$-2\pi \leq \delta_{k,\nu} \leq 2\pi, \ \forall \ k, \nu. \tag{3.41}$$

*Indices and sets*

$$et \in \mathcal{E}_t; \ h \in \mathcal{H}; \ h, i, j, \nu \in \mathcal{N}; \ i\text{-}j \in \mathcal{I}; \ k \in \mathcal{K}; \ \mathcal{H}, \mathcal{I} \subset \mathcal{N}. \tag{3.42}$$

### 3.5.2 Deterministic Model *vs.* Two-Stage Stochastic Model

In this section, a numerical example for transmission expansion in a vertically integrated industry is presented; it shows the advantages that a stochastic model has over a deterministic model. The VSS and the EVPI are also obtained. It is assumed that the generating capacity remains constant over the planning period. The system to be expanded is the classic test system proposed by Garver in 1970; it is shown in Figure 3.1 [30, 44].

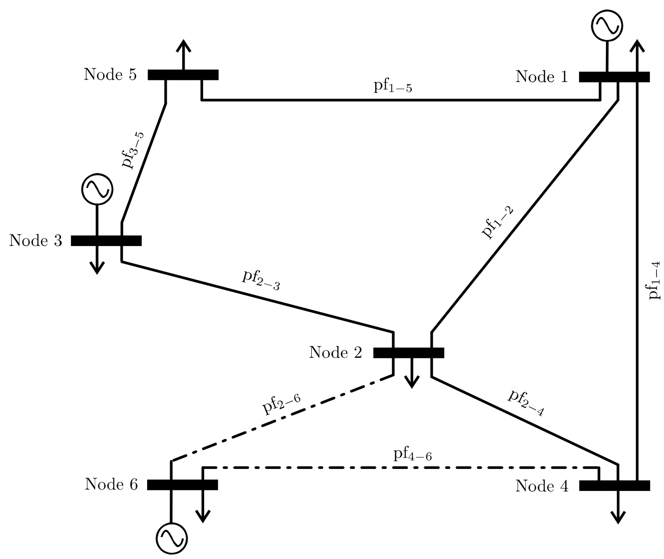


Figure 3.1: Six-node system to be expanded.

Three possible scenarios are considered: below average, average, and above average level of demand. It is assumed that one can add up to five new transmission lines between the pair of nodes that already have an interconnection. There are also two new rights-of-way between Node 6 and Node 2, and between Node 6 and Node 4 in which five new transmission lines can be built. Assume that the budget constraint is not binding. The expected value of the equivalent availability factor of the generating units is used. All the data is shown in Appendix C.

First, taking Node 1 as the slack[12] node, the deterministic transmission expansion

[12]Also known as the swing bus; a definition taken from [45] is: "One bus, known as *slack* or *swing bus*, is taken as reference where the magnitude and phase angle of the voltage are specified. This bus makes up the difference between the scheduled loads and generated power that are caused by the losses in the

problem shown in Equations (2.33)–(2.44) is solved[13] for each one of the three different foreseeable scenarios. Tables 3.8–3.13 show the results for the below average, average, and above average levels of demand. For the below average level of demand, one new line is built between Nodes 1 and 5, and 2 and 6; while two new lines are built between Nodes 4 and 6. For the average level of demand, two new transmission lines are built between Nodes 1 and 5, and 4 and 6; and one new transmission line between Nodes 3 and 5, and 2 and 6. For the above average level of demand, two new transmission lines are built between Nodes 1 and 5, and 4 and 6; and one new transmission line between Nodes 1 and 2, and 2 and 6.

Table 3.8: Deterministic, trans. exp., generation, six-node system, below.

| Scenario | Generation† | | |
|---|---|---|---|
| Below Average | 1 | 3 | 6 |
| | 370.0 | 0.0 | 370.0 |

† in MW; total cost: $210.569M.

Table 3.9: Deterministic, trans. exp., power flows, six-node system, below.

| Scenario | Power Flow† | | | |
|---|---|---|---|---|
| Below Average | $pf_{1-2}$ | $pf_{1-4}$ | $pf_{1-5}$ | $pf_{2-3}$ |
| | 63.6 | 37.4 | 178.9 | 31.1 |
| | $pf_{2-4}$ | $pf_{2-6}$ | $pf_{3-5}$ | $pf_{4-6}$ |
| | −7.5 | −100.0 | −68.9 | −180.0 |

† in MW; total cost: $210.569M.

network."

[13]The models in this section are implemented in the optimization software GAMS using the MINOS solver [42].

Table 3.10: Deterministic, trans. exp., generation, six-node system, average.

| Scenario | Generation† | | |
|---|---|---|---|
| Average | 1 | 3 | 6 |
| | 550.0 | 0.0 | 370.0 |

† in MW; total cost: $282.083M.

Table 3.11: Deterministic, trans. exp., power flows, six-node system, average.

| Scenario | Power Flow† | | | |
|---|---|---|---|---|
| Average | $pf_{1-2}$ | $pf_{1-4}$ | $pf_{1-5}$ | $pf_{2-3}$ |
| | 87.2 | 62.1 | 280.6 | −10.6 |
| | $pf_{2-4}$ | $pf_{2-6}$ | $pf_{3-5}$ | $pf_{4-6}$ |
| | 5.9 | −78.1 | −140.6 | −171.9 |

† in MW; total cost: $282.083M.

Since the average level of demand is most likely to happen, a decision maker might invest in the solution for the average level of demand. This expansion might be able to support the load for the below average demand scenario but might not support, at least not economically, the load for the above average demand scenario. Expanding the system for the solution of the above average level of demand, might support the load for the three different scenarios but at a high and uneconomical cost of $362.063 M. Taking a scenario analysis approach, for a given level of demand, one can calculate the worst that could happen—in terms of all the objectives—and then choose a solution that minimizes the value of the worst-case loss. This should single out some point that is optimal in a pessimistic minimax sense. An expansion project capable of meeting the highest foreseeable level of demand may turn out to be a quite expensive solution in the long run [46]. Once more, a better approach is to take into account the randomness of the stochastic variables. Tables 3.14 and 3.15 show the results for the stochastic model shown in Equations (3.31)–(3.42)

Table 3.12: Deterministic, trans. exp., generation, six-node system, above.

| Scenario | Generation† | | |
|---|---|---|---|
| Above Average | 1 | 3 | 6 |
| | 675.5 | 54.5 | 370.0 |

† in MW; total cost: $362.063M.

Table 3.13: Deterministic, trans. exp., power flows, six-node system, above.

| Scenario | Power Flow† | | | |
|---|---|---|---|---|
| Above Average | $\text{pf}_{1-2}$ | $\text{pf}_{1-4}$ | $\text{pf}_{1-5}$ | $\text{pf}_{2-3}$ |
| | 180.5 | 76.7 | 268.3 | 7.2 |
| | $\text{pf}_{2-4}$ | $\text{pf}_{2-6}$ | $\text{pf}_{3-5}$ | $\text{pf}_{4-6}$ |
| | 24.7 | −54.4 | −98.3 | −168.6 |

† in MW; total cost: $362.063M.

taking into consideration the randomness on the demand side. The expansion consists on the building of one transmission line between Nodes 1 and 2, two transmission lines between Nodes 1 and 5, and three transmission lines between Nodes 6 and 4.

It is easily seen that taking into account the randomness in the demand side produces a transmission expansion plan that supports the three foreseeable levels of demand but at an economical cost of $288.117 M. This implies savings of about $74 M with respect to the deterministic above average level of demand solution. Once more, the best solution that a decision maker can take is to use a stochastic model.

To obtain the VSS, the solution of the stochastic model is used. It is also needed to solve the stochastic model again but with the values for the new transmission added, $n_{i\text{-}j}$, fixed to the result obtained by the deterministic solution with the average level of demand. The EEV solution is shown in Tables 3.16 and 3.17.

From Equation (3.4), the VSS is

Table 3.14: Stochastic, trans. exp., generation, six-node system.

| Scenario | Generation† | | |
|---|---|---|---|
| Below Average | 1 | 3 | 6 |
| | 370.0 | 0.0 | 370.0 |
| Average | | | |
| | 543.6 | 6.4 | 370.0 |
| Above Average | | | |
| | 669.1 | 60.9 | 370.0 |

† in MW; total cost: $288.117M.

$$
\begin{aligned}
\text{EEV} &= \$291.6854 \text{ M}, && (3.43)\\
\text{VSS} &= \text{EEV} - \$288.117 \text{ M} \\
&= \$3.5684 \text{ M}. && (3.44)
\end{aligned}
$$

This means that, by considering the random variations, the investment in building new transmission lines is $288.117 M instead of $291.6854 M; which means that $3.5684 M are saved.

To compute the EVPI, it is needed to obtain first the WSS; the WSS is the mean value of all the deterministic solutions for each one of the foreseeable levels of demand.

From Equation (3.5), the expected value of perfect information is

$$
\begin{aligned}
\text{WSS} &= (0.3 \times \$210.569 \text{ M}) + (0.4 \times \$282.033 \text{ M}) + (0.3 \times \$362.063 \text{ M}) \\
&= \$284.623 \text{ M}, && (3.45)\\
\text{EVPI} &= \$288.117 - \text{WSS} \\
&= \$3.4942 \text{ M}. && (3.46)
\end{aligned}
$$

If it were possible to know the future demand perfectly, the cost would be $284.623 M

Table 3.15: Stochastic, trans. exp., power flows, six-node system.

| Scenario | Power Flow† | | | |
|---|---|---|---|---|
| Below Average | $pf_{1-2}$ | $pf_{1-4}$ | $pf_{1-5}$ | $pf_{2-3}$ |
| | 104.9 | −7.0 | 182.1 | 27.9 |
| Average | | | | |
| | 156.4 | 27.3 | 240.0 | 23.6 |
| Above Average | | | | |
| | 190.9 | 58.2 | 270.0 | −0.9 |
| Below Average | $pf_{2-4}$ | $pf_{3-5}$ | $pf_{4-6}$ | |
| | −63.0 | −72.1 | −280.0 | |
| Average | | | | |
| | −37.3 | −100.0 | −250.0 | |
| Above Average | | | | |
| | −8.2 | −100.0 | −220.0 | |

† in MW; total cost: $288.117M.

instead of $288.117 M, saving up to $3.4942 M. Since it is impossible to know the future demands in advance, the best that can be done is to take the SS as the best result.

These results show that the stochastic model, taking into account the randomness in the stochastic variables, is a good approximation since it is not too far from the result obtained by the WSS.

Table 3.16: EEV solution, trans. exp., generation, six-node system, average.

| Scenario | Generation† | | |
|---|---|---|---|
| Below Average | 1 | 3 | 6 |
| | 370.0 | 0.0 | 370.0 |
| Average | | | |
| | 550.0 | 0.0 | 370.0 |
| Above Average | | | |
| | 574.5 | 155.5 | 370.0 |

† in MW; total cost: $291.685M.

Table 3.17: EEV solution, trans. exp., power flows, six-node system, average.

| Scenario | Power Flow† | | | |
|---|---|---|---|---|
| Below Average | $pf_{1-2}$ | $pf_{1-4}$ | $pf_{1-5}$ | $pf_{2-3}$ |
| | 50.1 | 30.8 | 199.2 | 10.8 |
| Average | | | | |
| | 87.2 | 62.1 | 280.6 | −10.6 |
| Above Average | | | | |
| | 97.1 | 80.0 | 247.4 | −72.9 |
| Below Average | $pf_{2-4}$ | $pf_{2-6}$ | $pf_{3-5}$ | $pf_{4-6}$ |
| | −3.9 | −96.8 | −89.2 | −183.2 |
| Average | | | | |
| | 5.9 | −78.1 | −140.6 | −171.9 |
| Above Average | | | | |
| | 22.9 | −52.9 | −77.4 | −167.1 |

† in MW; total cost: $291.685M.

### 3.5.3 Two-Stage Model with Risk

A risk parameter can be incorporated into the objective function of the two-stage stochastic model for transmission expansion using the mean-variance Markowitz theory. From Equation (3.29), the variance of $z$ is

$$\sigma_z^2 = \sum_k pr_k \left( \sum_{et,h} q_{et} g_{k,et,h} \right)^2 - \left( \sum_{k,et,h} pr_k q_{et} g_{k,et,h} \right)^2 . \tag{3.47}$$

Using Equations (3.30) and (3.47), the new objective function of the two-stage stochastic model with fixed recourse that incorporates the risk parameter is

$$\begin{aligned} \min \sum_{i\text{-}j} c_{i\text{-}j} n_{i\text{-}j} + \sum_{k,et,h} pr_k q_{et} g_{k,et,h} \\ + \theta_r \left\{ \sum_k pr_k \left( \sum_{et,h} q_{et} g_{k,et,h} \right)^2 - \left( \sum_{k,et,h} pr_k q_{et} g_{k,et,h} \right)^2 \right\} ; \end{aligned} \tag{3.48}$$

the constraints remain as in Equations (3.32)–(3.42).

Just like before, the risk parameter $\theta_r$ weighs how important the minimization of the variance as a risk measure is. Table 3.18 shows some results as $\theta_r$ varies. As the risk aversion increases, the standard deviation decreases, and the overall cost increases.

Table 3.18: Annualized generating cost, $\theta_r$ varies, trans. exp., six-node system.

| Risk Parameter | Mean† | Std. Deviation† | Overall Cost‡ |
|---|---|---|---|
| $\theta_r = 0.00$ | 270.617 | 56.815 | 288.117 |
| $\theta_r = 0.01$ | 279.745 | 46.050 | 297.245 |
| $\theta_r = 0.02$ | 289.146 | 36.2904 | 306.646 |
| $\theta_r = 0.03$ | 305.674 | 25.458 | 323.174 |
| $\theta_r = 0.07$ | 327.896 | 10.910 | 347.646 |
| $\theta_r = 0.08$ | 329.980 | 9.547 | 349.730 |

† in \$M-y; ‡ Investment cost plus annualized generating cost, in \$M-y.

### 3.5.4 Two-Stage Model with Risk and Probabilistic Constraints

In the transmission expansion problem, not only the equivalent availability for each generating plant can be considered as random; the transmission capacity factor of the transmission lines is random too. The actual capacity of a transmission line depends on exogenous environmental parameters and other contingencies such as random outages and discretional security constraints implemented by the system operator. Consider Path 15 that connects Northern and Southern California. The capacity of this link varies between 2,600 MW and 3,950 MW depending upon ambient temperature and remedial action schemes that are in place to respond to unanticipated outages of generating plants and other transmission lines [13].

To include these two random parameters into the two-stage stochastic model, three probabilistic constraints are added; one for the equivalent availability of the generating units $\alpha_{et}$, and two for the random transmission capacity factor $\alpha_{i\text{-}j}$. As before, it is assumed that the mean and the variance of these parameters are known and that they are standard normally distributed random variables.

*Probabilistic transmission line capacity constraints*

$$f_{k,i\text{-}j} \leq \alpha_{i\text{-}j}(n^0_{i\text{-}j} + n_{i\text{-}j})\overline{f}_{i\text{-}j} - \beta_t \left\{ \sigma^2_{\alpha_{i\text{-}j}} \left( (n^0_{i\text{-}i} + n_{i\text{-}j})\overline{f}_{i\text{-}j} \right)^2 \right\}^{\frac{1}{2}}, \ \forall\ k, i\text{-}j, \tag{3.49}$$

$$-f_{k,i\text{-}j} \leq \alpha_{i\text{-}j}(n^0_{i\text{-}j} + n_{i\text{-}j})\overline{f}_{i\text{-}j} - \beta_t \left\{ \sigma^2_{\alpha_{i\text{-}j}} \left( (n^0_{i\text{-}i} + n_{i\text{-}j})\overline{f}_{i\text{-}j} \right)^2 \right\}^{\frac{1}{2}}, \ \forall\ k, i\text{-}j. \tag{3.50}$$

*Probabilistic derated capacity constraint*

$$g_{k,et,h} - \alpha_{et} y_{et,h} + \beta_g \left\{ \sigma^2_{\alpha_{et}} (y_{et,h})^2 \right\}^{\frac{1}{2}} \leq 0, \ \forall\ k, et, h. \tag{3.51}$$

In Equations (3.49)–(3.51), $\beta_g$ and $\beta_t$ are the $\alpha$-quantile of the standard normally distributed random variables $\alpha_{et}$ and $\alpha_{i\text{-}j}$, respectively. The parameters $\beta_g$ and $\beta_t$ are chosen by the decision maker. Equations (3.49)–(3.51) are linearized next:

$$f_{k,i\text{-}j} \leq \alpha_{i\text{-}j}(n^0_{i\text{-}j} + n_{i\text{-}j})\overline{f}_{i\text{-}j} - \beta_t \sigma_{\alpha_{i\text{-}j}} (n^0_{i\text{-}j} + n_{i\text{-}j})\overline{f}_{i\text{-}j}, \forall\ k, i\text{-}j, \tag{3.52}$$

$$-f_{k,i\text{-}j} \leq \alpha_{i\text{-}j}(n^0_{i\text{-}j} + n_{i\text{-}j})\overline{f}_{i\text{-}j} - \beta_t \sigma_{\alpha_{i\text{-}j}} (n^0_{i\text{-}j} + n_{i\text{-}j})\overline{f}_{i\text{-}j}, \forall\ k, i\text{-}j, \tag{3.53}$$

$$g_{k,et,h} - \alpha_{et} y_{et,h} + \beta_g \sigma_{\alpha_{et}} y_{et,h} \leq 0, \ \forall\ k, et, h. \tag{3.54}$$

Table 3.19 shows some results when $\theta_r$ is fixed to a value of 0.02 and $\beta_t$ is allowed to vary. Table 3.20 shows some results when $\theta_r$ is fixed to a value of 0.02 and $\beta_g$ is allowed to vary. Notice that as the probability of meeting the chance constraints increases, the investment and annualized generating costs also increase.

Table 3.19: Annualized generating cost, $\beta_t$ varies, trans. exp., six-node system.

| Probability | Mean† | Standard Deviation † | Overall Cost‡ |
|---|---|---|---|
| $\approx 76\%; \beta_t = 0.7$ | 291.532 | 37.632 | 309.032 |
| $\approx 84\%; \beta_t = 1.0$ | 296.100 | 38.188 | 313.600 |
| $\approx 90\%; \beta_t = 1.3$ | 289.146 | 36.290 | 310.646 |
| $\approx 96\%; \beta_t = 1.74$ | 289.146 | 36.290 | 310.646 |

† in \$M-y; ‡ Investment cost plus annualized generating cost, in \$M-y.

Table 3.20: Annualized generating cost, $\beta_g$ varies, trans. exp., six-node system.

| Probability | Mean† | Standard Deviation † | Overall Cost‡ |
|---|---|---|---|
| $\approx 76\%; \beta_g = 0.7$ | 320.122 | 40.438 | 332.372 |
| $\approx 84\%; \beta_g = 1.0$ | 331.632 | 45.708 | 345.482 |
| $\approx 90\%; \beta_g = 1.3$ | 351.082 | 38.188 | 367.182 |
| $\approx 96\%; \beta_g = 1.74$ | 369.006 | 40.857 | 382.106 |

† in \$M-y; ‡ Investment cost plus annualized generating cost, in \$M-y.

For different values of $\theta_r$, and keeping $\beta_g$ and $\beta_t$ fixed to a value of 1.3 (at least a 90% probability the chance constraints are met), the mean and the standard deviation of the annualized generating cost are obtained. The results are presented in Table 3.21. Notice that as $\theta_r$ increases, the overall cost increases and the variance of the annualized generation cost decreases.

Table 3.21: Annualized generating cost, $\theta_r$ varies, trans. exp.,six-node system.

| Risk Parameter | Mean† | Standard Deviation† | Overall Cost‡ |
|---|---|---|---|
| $\theta_r = 0.00$ | 144.704 | 32.043 | 260.084 |
| $\theta_r = 0.03$ | 154.502 | 16.428 | 269.882 |
| $\theta_r = 0.04$ | 154.366 | 19.094 | 273.856 |
| $\theta_r = 0.05$ | 160.200 | 15.275 | 279.690 |
| $\theta_r = 0.06$ | 164.089 | 12.729 | 283.529 |
| $\theta_r = 0.10$ | 171.866 | 7.637 | 291.356 |

† in \$M-y; ‡ Investment cost plus annualized generating cost, in \$M-y.

## 3.6 Stochastic Generation and Transmission Expansion

In this section, the stochastic generation and transmission expansion models are combined into one single model. First, a two-stage stochastic model is formulated in order to account for the randomness in the demand levels. After that, a risk parameter is introduced into the objective function using the mean-variance Markowitz theory. Finally, probabilistic constraints are added to account for the randomness in the equivalent availability factors and in the transmission capacity factors. The most complete model is a mixed-integer nonlinear optimization problem. Two systems are solved to exemplify the models: a six- and a 21-node system.

### 3.6.1 Two-Stage Model

In this two-stage stochastic model, the first-stage decisions are the investment in new generation and transmission while the second stage decision is the annual estimate of generation in order to minimize the annualized generation cost of the existing and possible new generating plants based on the expansion made. Consequently, the second stage objective can be thought as the minimization of the annualized production cost.

Redefine $z$ as

$$z = \sum_{m} cf_m \left( \sum_{et,h} q_{et} g_{m,et,h} + \sum_{nt,p} q_{nt} g_{m,nt,p} \right). \tag{3.55}$$

The expected value of $z$ is

$$\begin{aligned} E\{z\} &= \sum_{k} pr_k z_k \\ &= \sum_{k} pr_k \left( \sum_{m} cf_m \left( \sum_{et,h} q_{et} g_{m,et,h} + \sum_{nt,p} q_{nt} g_{m,nt,p} \right) \right) \\ &= \sum_{k,m} pr_k cf_m \left( \sum_{et,h} q_{et} g_{k,m,et,h} \sum_{nt,p} q_{nt} g_{k,m,nt,p} \right). \end{aligned} \tag{3.56}$$

In the two-stage stochastic model presented next, the investment cost in new transmission lines and in new generating units as well as the expected value of the generation cost of existing and newly installed generating capacity are minimized. The new objective function is shown in Equation (3.57). The budget constraint, Equation (2.46), and the upper and lower limit constraint for the new circuits to be added in each right-of-way, Equation (2.54), remain as in the deterministic model. All the other constraints are modified to take into account all the different foreseeable scenarios of demand $k \in \mathcal{K}$.

*Objective function*

$$\min \quad \sum_{i\text{-}j} c_{i\text{-}j} n_{i\text{-}j} + \sum_{nt,p} r_{nt} x_{nt} n_{nt,p} + \sum_{k,m} pr_k cf_m \left( \sum_{nt,p} q_{nt} g_{k,m,nt,p} + \sum_{et,h} q_{et} g_{k,m,et,h} \right). \tag{3.57}$$

*Derated capacity constraints*

$$\sum_{m} (g_{k,m,nt,p}) - \alpha_{nt} x_{nt} n_{nt,p} \leq 0, \ \forall \ k, nt, p, \tag{3.58}$$

$$\sum_{m} (g_{k,m,et,h}) - \alpha_{et} y_{et,h} \leq 0, \ \forall \ k, et, h. \tag{3.59}$$

*Nodal balance constraint*

$$\boldsymbol{S f_{k,m}} + \boldsymbol{g_{k,m}} = \boldsymbol{d_{k,m}}, \ \forall \ k, m. \tag{3.60}$$

*Power flow constraint*

$$f_{k,m,i\text{-}j} - \Gamma_{i\text{-}j}(n^0_{i\text{-}j} + n_{i\text{-}j})(\delta_{k,m,i} - \delta_{k,m,j}) = 0, \ \forall \ k, m, i\text{-}j. \tag{3.61}$$

*Transmission line capacity constraints*

$$f_{k,m,i\text{-}j} \leq (n^0_{i\text{-}j} + n_{i\text{-}j})\overline{f}_{i\text{-}j}, \ \forall \ k, m, i\text{-}j, \tag{3.62}$$

$$-f_{k,m,i\text{-}j} \leq (n^0_{i\text{-}j} + n_{i\text{-}j})\overline{f}_{i\text{-}j}, \ \forall \ k, m, i\text{-}j. \tag{3.63}$$

*Stability constraint*

$$-\frac{1}{4}\pi \leq (\delta_{k,m,i} - \delta_{k,m,j}) \leq \frac{1}{4}\pi, \ \forall \ k, m, i\text{-}j. \tag{3.64}$$

*Upper and lower limit constraints*

$$g_{k,m,et,h} \geq 0, \ \forall \ k, m, et, h, \tag{3.65}$$

$$g_{k,m,nt,p} \geq 0, \ \forall \ k, m, nt, p, \tag{3.66}$$

$$-2\pi \leq \delta_{k,m,\nu} \leq 2\pi, \ \forall \ k, m, \nu. \tag{3.67}$$

*Indices and sets*

$$\begin{gathered} et \in \mathcal{E}_t;\ h \in \mathcal{H};\ h, i, j, p, \nu \in \mathcal{N};\ i\text{-}j \in \mathcal{I}; \\ k \in \mathcal{K};\ m \in \mathcal{M};\ nt \in \mathcal{N}_t;\ p \in \mathcal{P};\ \mathcal{H}, \mathcal{I}, \mathcal{P} \subset \mathcal{N}. \end{gathered} \tag{3.68}$$

### 3.6.2 Deterministic Model *vs.* Two-Stage Stochastic Model

This section compares the deterministic models *vs.* the stochastic models by means of a six- and a 21-node system.

#### A Six-Node System

Consider first a six-node system. It is a modification of the system shown in Section 3.5.2. In the system shown in Figure 3.2, one can add up to five new transmission lines between the pair of nodes that already have an interconnection. There are also two new rights-of-way between Node 6 and Node 2, and between Node 6 and Node 4 in which up to five new transmission lines can be built. There are three generators located at Nodes 1, 3, and 6, all of which are of different technologies. New generation can be added at Nodes 2 and 4, and there are three different technologies to choose from. The parameters for the existing and new transmission lines, as well as for the installed and new generating capacity are given in Appendix C. It is assumed that the budget constraint is not binding and Node 1 is taken as the swing bus.

At every node there are three different operating modes: i) base, ii) mid, and iii) peak. Without any loss of generality, it is assumed that each operating mode at every node has the same duration. Each operating mode has three forecasted levels of demand: i) below average, ii) average, and iii) above average. These forecasted levels of demand are known to happen with 30%, 40%, and 30% probability, respectively. All the pertinent data appears in Appendix C.

By solving[14] the deterministic model shown in Equations (2.45)–(2.58) for each one of the three foreseeable levels of demand, one can see that the cost varies depending on which level of demand is used in the investment plan. For instance, for the below average level

[14] All models presented in this section are implemented in the optimization software GAMS using the SBB solver.

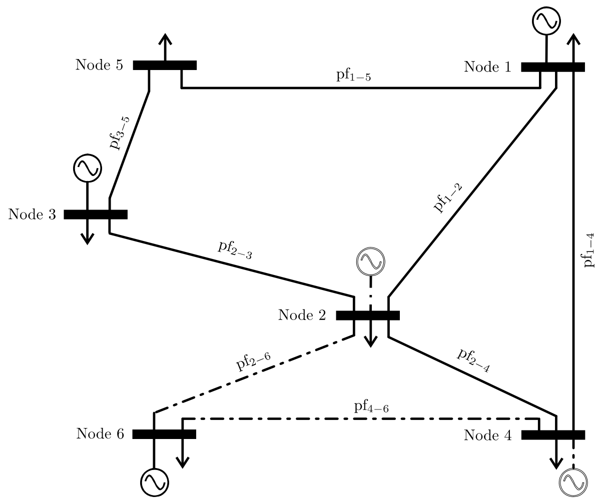


Figure 3.2: Six-node system to be expanded.

of demand the total cost is \$151.161 M, for the average level of demand the total cost is \$202.272 M, and for the above average level of demand the total cost is \$250.074 M.

The other approach to solve this problem, the stochastic approach, is to take into account the probabilities of all the possible different levels of demand (possible scenarios). The solution, shown in Tables 3.22 and 3.23, is obtained by solving the model shown in Equations (2.46), (2.54), and (3.57)–(3.68). The total cost is now \$209.674 M; by building a new transmission line between Nodes 1 and 5, 2 and 3, 2 and 6, 3 and 5, and one new generator of technology two at Node 4, the three foreseeable levels of demand are satisfied and \$40.4 M are saved with respect to the result obtained for above average level of demand.

To obtain the EVPI, it is needed first to compute the WSS, which is the mean value of all the deterministic solutions for each one of the foreseeable levels of demand. This is shown in Equation (3.69).

$$\begin{aligned} \text{WSS} &= (0.3 \times \$151.161\text{M}) + (0.4 \times \$202.272\text{M}) + (0.3 \times \$250.074\text{M}) \\ &= \$201.279\text{M}. \end{aligned} \tag{3.69}$$

From Equation (3.5), the EVPI is \$8.395 M. If it were possible to know in advance the future demand, the cost would be \$201.279 M instead of \$209.674 M, saving up to \$8.395

Table 3.22: Stochastic, gen. & trans. exp., generation, six-node system.

| Operating Mode† | Generator | | | |
|---|---|---|---|---|
| Below Average | 1 | 3 | 4 | 6 |
| B‡ | 448.9 | 31.1 | 0.0 | 0.0 |
| B‡ + M⋆ | 460.0 | 100.0 | 100.0 | 0.0 |
| B‡ + M⋆ + P∗ | 460.0 | 150.0 | 110.0 | 20.0 |
| Average | | | | |
| B‡ | 462.5 | 69.3 | 8.2 | 0.0 |
| B‡ + M⋆ | 462.5 | 124.0 | 183.5 | 10.0 |
| B‡ + M⋆ + P∗ | 462.5 | 206.7 | 230.8 | 20.0 |
| Above Average | | | | |
| B‡ | 462.5 | 96.7 | 40.8 | 0.0 |
| B‡ + M⋆ | 462.5 | 275.7 | 131.8 | 30.0 |
| B‡ + M⋆ + P∗ | 462.5 | 289.5 | 289.4 | 58.5 |

† in MW; ‡ Base, ⋆ Mid, and ∗ Peak.

M. Since it is impossible to know the future demand in advance, the best that can be done is to take the SS as the best result. These results show that the stochastic model, taking into account the probabilities of all the foreseeable scenarios, is a good representation of the uncertainties faced by the investment plan since the result obtained by the SS is not too far from the result obtained by the WSS.

To compute the VSS, one first needs to obtain the EEV. When trying to compute the EEV, the problem becomes infeasible. Since the VSS is the cost of ignoring uncertainty, one can add a set of slack variables in the nodal balance constraint to provide for any generation deficiency. Also, these slack variables need to be included in the objective function; each with a weighing coefficient that is at least greater than the most expensive annualized variable generating cost. This is equivalent to assume that there is generating capacity available at each bus ready to be used in the presence of a contingency but at a greater cost. Hence, the EEV can be redefined as the *solution of the stochastic problem, but considering the number of new generating units and new transmission lines added fixed to the results obtained by the deterministic model for the average level of demand plus the*

Table 3.23: Stochastic, gen. & trans. exp., power flows, six-node system.

| Operating Mode† | Power Flow | | | | | | |
|---|---|---|---|---|---|---|---|
| Below Average | $pf_{1-2}$ | $pf_{1-4}$ | $pf_{1-5}$ | $pf_{2-3}$ | $pf_{2-4}$ | $pf_{2-6}$ | $pf_{3-5}$ |
| B‡ | 97.8 | 71.6 | 200.0 | −71.1 | 8.9 | 80.0 | −120.0 |
| B‡ + M⋆ | 100.0 | 80.0 | 190.0 | −110.0 | 20.0 | 90.0 | −100.0 |
| B‡ + M⋆ + P∗ | 100.0 | 80.0 | 190.0 | −130.0 | 20.0 | 70.0 | −80.0 |
| Average | | | | | | | |
| B‡ | 99.8 | 72.7 | 200.0 | −89.3 | 9.2 | 90.0 | −110.0 |
| B‡ + M⋆ | 98.5 | 54.0 | 200.0 | −104.0 | −17.5 | 100.0 | −90.0 |
| B‡ + M⋆ + P∗ | 99.2 | 43.2 | 200.0 | −136.7 | −34.2 | 100.0 | −60.0 |
| Above Average | | | | | | | |
| B‡ | 99.2 | 63.3 | 200.0 | −96.7 | −4.2 | 100.0 | −100.0 |
| B‡ + M⋆ | 91.9 | 80.0 | 160.6 | −176.3 | 28.1 | 100.0 | −30.6 |
| B‡ + M⋆ + P∗ | 91.5 | 28.8 | 192.2 | −151.7 | −48.3 | 91.5 | −22.2 |

† in MW; ‡ Base, ⋆ Mid, and ∗ Peak.

*cost of providing for any energy deficiency.* In this numerical example, there is a deficiency of 600 MW for the base load mode and of 34.2 MW for the peak load mode of the above average level of demand. Under the assumption that the annualized variable generating cost of providing for any energy deficiency at every node is 1 \$M/MW-y, the EEV is \$227.6820M. Therefore, form Equation (3.4), the VSS is \$18.008M.

**A 21-Node System**

This twenty-one-node system is a modification of the reliability test system proposed by the IEEE Reliability Test System Task Force [47] in 1979. In the system shown in Figure 3.3, one can add two new transmission lines between Nodes 2 and 4, and Nodes 2 and 6; three new transmission lines between Nodes 3 and 13; and six new transmission lines between Nodes 9 and 11, 9 and 12, 10 and 11, and 10 and 21. There is a mix of 13 generators of four different technologies. New generation can be added at Nodes 2, 11, 13, 14, and 21, and there are four different technologies to choose from. The parameters for the existing and new transmission lines, as well as for the installed and new generating capacity are given in Appendix C.

Like in the six-node system, at every node there are three different operating modes, each of which has three forecasted levels of demand. These forecasted levels of demand happen with 30%, 40%, and 30% probability, respectively. Appendix C gives all pertinent data.

Solving the deterministic model for each one of the three foreseeable levels of demand, one gets a total cost of \$483.498 M, \$664.737 M, and \$815.467 M for the below average, average, and above average levels of demand, respectively.

Using the stochastic approach, one gets the solution shown in Tables 3.24 and 3.25 at a total cost of \$679.8 M. The expansion consists on the building of a new transmission line between Nodes 2 and 4, 2 and 6, 9 and 11, 10 and 21, one new generator of technology one at Node 2, one new generator of technology three at Node 13, and two new generators of technology four at Node 2. The three foreseeable levels of demand are satisfied and \$135.667 M are saved with respect to the result obtained for above average level of demand. The expected value of perfect information is \$24.2157 M.

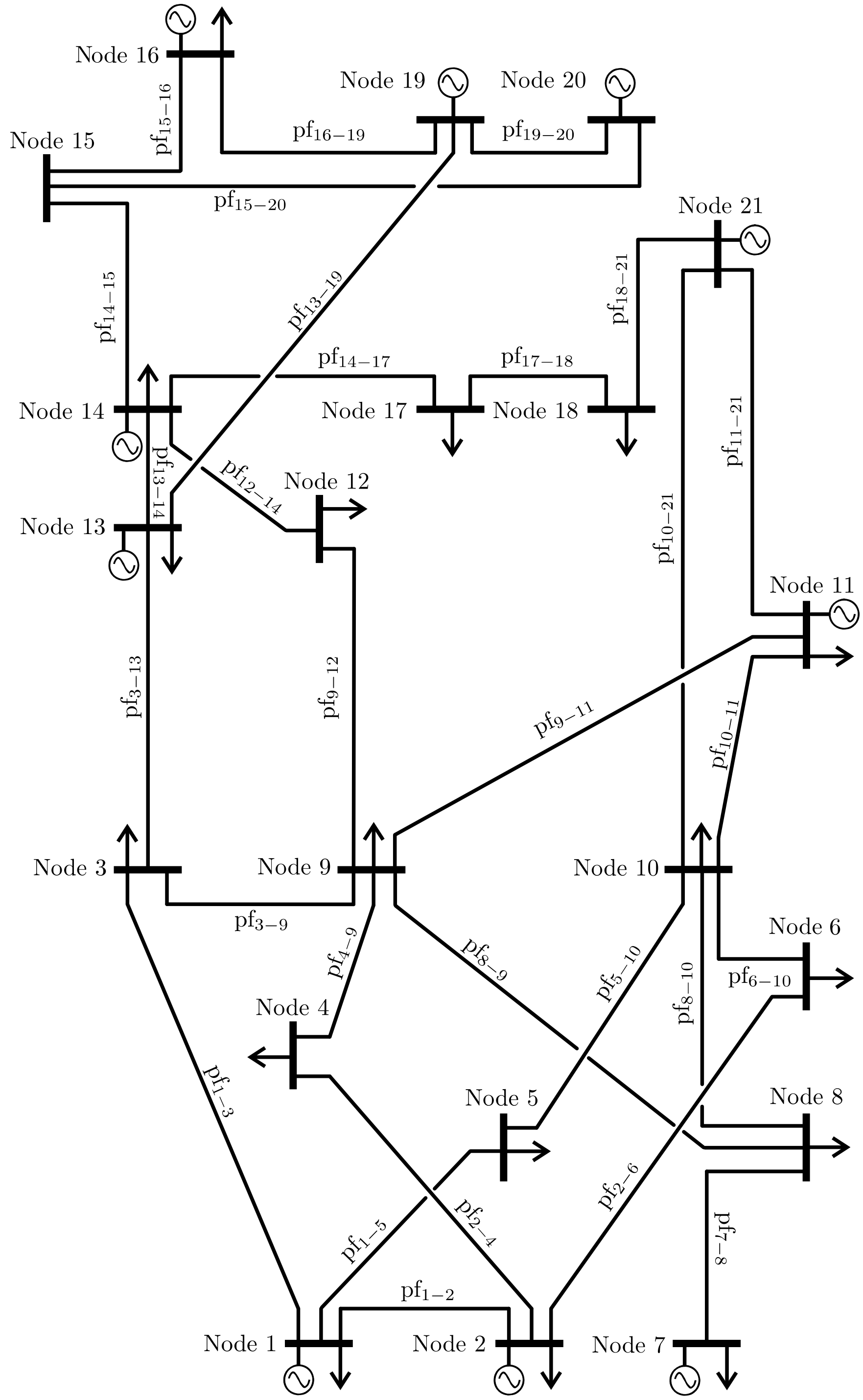


Figure 3.3: 21-node system to be expanded.

Table 3.24: Stochastic, gen. & trans. exp., generation, 21-node system.

| Operating Mode† | Node | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Below Average | 1 | 2 | 7 | 11 | 13 | 14 | 16 | 19 | 20 | 21 |
| B‡ | 0.0 | 421.0 | 0.0 | 0.0 | 0.0 | 0.0 | 181.5 | 370.0 | 277.5 | 0.0 |
| B‡ + M⋆ | 138.8 | 503.8 | 0.0 | 0.0 | 0.0 | 28.4 | 370.0 | 370.0 | 277.5 | 311.6 |
| B‡ + M⋆ + P∗ | 138.8 | 688.8 | 211.6 | 0.0 | 138.7 | 138.7 | 370.0 | 370.0 | 277.5 | 515.9 |
| Average | | | | | | | | | | |
| B‡ | 19.4 | 546.2 | 0.0 | 0.0 | 0.0 | 0.0 | 286.9 | 370.0 | 277.5 | 0.0 |
| B‡ + M⋆ | 138.8 | 666.0 | 0.0 | 0.0 | 107.2 | 0.0 | 370.0 | 370.0 | 277.5 | 470.6 |
| B‡ + M⋆ + P∗ | 167.1 | 693.8 | 289.5 | 393.1 | 138.7 | 138.7 | 370.0 | 370.0 | 277.5 | 581.6 |
| Above Average | | | | | | | | | | |
| B‡ | 82.2 | 625.3 | 0.0 | 0.0 | 0.0 | 0.0 | 370.0 | 370.0 | 277.5 | 0.0 |
| B‡ + M⋆ | 138.7 | 689.0 | 145.8 | 0.0 | 138.8 | 138.8 | 370.0 | 370.0 | 277.5 | 491.4 |
| B‡ + M⋆ + P∗ | 177.3 | 723.4 | 289.5 | 561.7 | 529.8 | 138.8 | 370.0 | 370.0 | 277.5 | 495.1 |

† in MW; ‡ Base, ⋆ Mid, and ∗ Peak.

Table 3.25: Stochastic, gen. & trans. exp., power flows, 21-node system.

| Operating Mode† | Power Flow | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Below Average | $pf_{1-2}$ | $pf_{1-5}$ | $pf_{1-3}$ | $pf_{2-4}$ | $pf_{2-6}$ | $pf_{3-9}$ | $pf_{3-13}$ | $pf_{4-9}$ | $pf_{5-10}$ | $pf_{6-10}$ | $pf_{7-8}$ | $pf_{8-9}$ |
| B‡ | −158.9 | 89.6 | 21.8 | 101.1 | 118.6 | 56.3 | −113.2 | 68.6 | 58.3 | 58.6 | −55.0 | −69.3 |
| B‡ + M⋆ | −121.3 | 126.4 | 57.6 | 155.2 | 159.3 | 56.2 | −124.6 | 103.2 | 76.4 | 63.3 | −88.0 | −99.7 |
| B‡ + M⋆ + P∗ | −200.0 | 147.3 | 83.1 | 198.8 | 193.1 | 43.6 | −140.0 | 124.7 | 76.1 | 56.3 | 86.2 | −31.6 |
| Average | | | | | | | | | | | | |
| B‡ | −200.0 | 119.8 | 42.6 | 138.5 | 156.7 | 61.5 | −113.4 | 99.5 | 82.3 | 84.7 | −66.0 | −81.2 |
| B‡ + M⋆ | −188.4 | 152.7 | 83.3 | 200.0 | 196.0 | 56.5 | −124.4 | 137.6 | 92.7 | 80.8 | −105.6 | −115.0 |
| B‡ + M⋆ + P∗ | −200.0 | 137.4 | 99.7 | 192.7 | 184.7 | −7.2 | −108.5 | 103.8 | 51.9 | 20.6 | 139.0 | −21.0 |
| Above Average | | | | | | | | | | | | |
| B‡ | −200.0 | 151.1 | 65.6 | 174.3 | 192.3 | 65.9 | −109.0 | 129.4 | 108.0 | 109.5 | −75.9 | −91.6 |
| B‡ + M⋆ | −198.3 | 151.2 | 81.1 | 200.0 | 196.8 | 51.2 | −144.0 | 128.2 | 82.2 | 64.3 | 24.4 | −61.0 |
| B‡ + M⋆ + P∗ | −198.4 | 149.3 | 77.0 | 191.2 | 200.0 | 19.5 | −190.3 | 88.9 | 51.0 | 11.2 | 116.4 | −55.4 |

| Operating Mode† | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Below Average | $pf_{8-10}$ | $pf_{9-11}$ | $pf_{9-12}$ | $pf_{10-11}$ | $pf_{10-21}$ | $pf_{11-21}$ | $pf_{12-14}$ | $pf_{13-14}$ | $pf_{13-19}$ | $pf_{14-15}$ | $pf_{14-17}$ | $pf_{15-16}$ |
| B‡ | −60.7 | 79.3 | −100.0 | 9.9 | −38.7 | −27.1 | −185.0 | 68.9 | −320.8 | −361.9 | 202.0 | −209.7 |
| B‡ + M⋆ | −108.3 | 31.5 | −93.7 | 45.6 | −150.2 | −108.9 | −229.7 | 17.6 | −364.3 | −419.2 | 165.6 | −268.4 |
| B‡ + M⋆ + P∗ | −53.2 | 20.9 | −58.2 | 85.4 | −200.0 | −158.7 | −252.0 | 12.7 | −330.3 | −353.7 | 153.5 | −203.2 |
| Average | | | | | | | | | | | | |
| B‡ | −74.8 | 88.3 | −100.0 | 21.7 | −31.5 | −29.5 | −202.0 | 67.7 | −347.6 | −411.3 | 224.5 | −259.0 |
| B‡ + M⋆ | −134.6 | 5.0 | −72.3 | 70.5 | −194.9 | −147.6 | −235.5 | 57.8 | −341.4 | −395.3 | 133.6 | −243.3 |
| B‡ + M⋆ + P∗ | −45.2 | −178.2 | 45.5 | −5.3 | −200.0 | −108.8 | −187.0 | −34.7 | −314.6 | −302.7 | 100.0 | −153.7 |
| Above Average | | | | | | | | | | | | |
| B‡ | −87.8 | 94.9 | −96.3 | 34.4 | −22.0 | −31.2 | −213.6 | 67.1 | −367.5 | −448.1 | 241.2 | −295.8 |
| B‡ + M⋆ | −80.2 | 23.2 | −73.1 | 78.5 | −200.0 | −154.9 | −260.8 | 21.0 | 332.6 | 361.9 | 164.3 | −211.1 |
| B‡ + M⋆ + P∗ | −64.2 | −200.0 | 13.1 | −69.4 | −200.0 | −73.5 | −254.3 | 164.6 | −261.6 | −295.7 | 207.1 | −140.8 |

| Operating Mode† | | | | | |
|---|---|---|---|---|---|
| Below Average | $pf_{15-20}$ | $pf_{16-19}$ | $pf_{17-18}$ | $pf_{18-21}$ | $pf_{19-20}$ |
| B‡ | −152.2 | −174.5 | 122.0 | 65.8 | −125.3 |
| B‡ + M⋆ | −150.8 | −132.4 | 37.6 | −52.4 | −126.7 |
| B‡ + M⋆ + P∗ | −150.5 | −166.7 | −28.9 | −157.2 | −127.0 |
| Average | | | | | |
| B‡ | −152.3 | −147.6 | 128.5 | 61.0 | −125.2 |
| B‡ + M⋆ | −152.0 | −154.1 | −20.0 | −128.0 | −125.5 |
| B‡ + M⋆ + P∗ | −149.0 | −183.9 | −118.9 | −272.8 | −128.5 |
| Above Average | | | | | |
| B‡ | −152.4 | −127.6 | 130.8 | 53.2 | −125.1 |
| B‡ + M⋆ | −150.8 | −164.1 | −12.3 | −136.5 | −126.7 |
| B‡ + M⋆ + P∗ | −155.0 | −230.9 | −44.7 | −221.6 | −122.5 |

† in MW; ‡ Base, ⋆ Mid, and ∗ Peak.

### 3.6.3 Two-Stage Model with Risk

To minimize the risk faced in the transmission and generation expansion problem, the interest is focused in the minimization of the investment cost in new transmission and in new generation, the expected annualized production cost, and in the minimization of the variance of the annualized production cost.

The variance of $z$, as defined in (3.55), is

$$\begin{aligned}
\sigma_z^2 &= E\{z^2\} - E^2\{z\} \\
&= \sum_k pr_k \left( \sum_m cf_m \left( \sum_{et,h} q_{et} g_{k,m,et,h} + \sum_{nt,p} q_{nt} g_{k,m,nt,p} \right) \right)^2 \\
&\quad - \left( \sum_{k,m} pr_k cf_m \left( \sum_{et,h} q_{et} g_{k,m,et,h} + \sum_{nt,p} q_{nt} g_{k,m,nt,p} \right) \right)^2 .
\end{aligned} \tag{3.70}$$

The objective function of the two-stage stochastic model that incorporates the variance and the risk parameter is shown in Equation (3.71). All the constraints remain the same as in Equations (2.46), (2.54), and (3.58)–(3.68).

$$\begin{aligned}
\min \quad & \sum_{i\text{-}j} c_{i\text{-}j} n_{i\text{-}j} + \sum_{nt,p} r_{nt} x_{nt} n_{nt,p} \\
& + \sum_{k,m} pr_k cf_m \left( \sum_{nt,p} q_{nt} g_{k,m,nt,p} + \sum_{et,h} q_{et} g_{k,m,et,h} \right) \\
& + \theta_r \left\{ \sum_k pr_k \left( \sum_m cf_m \left( \sum_{et,h} q_{et} g_{k,m,et,h} + \sum_{nt,p} q_{nt} g_{k,m,nt,p} \right) \right)^2 \right. \\
& \left. - \left( \sum_{k,m} pr_k cf_m \left( \sum_{et,h} q_{et} g_{k,m,et,h} + \sum_{nt,p} q_{nt} g_{k,m,nt,p} \right) \right)^2 \right\} .
\end{aligned} \tag{3.71}$$

### 3.6.4 Two-Stage Model with Risk and Probabilistic Constraints

Just like before, the derated capacity constraints can be modified to take into account a random availability factor for the generating plants by means of a chance (probabilistic) constraint. In like manner, the transmission line capacity constraints can be modified to take into account a random transmission capacity factor. This is shown next.

*Probabilistic derated capacity constraints*

$$\sum_m (g_{k,m,nt,p}) - \alpha_{nt} x_{nt} n_{nt,p} + \beta_g \left\{ \sigma^2_{\alpha_{nt}} \left( x_{nt} n_{nt,p} \right)^2 \right\}^{\frac{1}{2}} \leq 0, \; \forall \; k, nt, p, \tag{3.72}$$

$$\sum_m (g_{k,m,et,h}) - \alpha_{et} y_{et,h} + \beta_g \left\{ \sigma^2_{\alpha_{et}} \left( y_{et,h} \right)^2 \right\}^{\frac{1}{2}} \leq 0, \; \forall \; k, et, h. \tag{3.73}$$

*Probabilistic transmission line capacity constraints*

$$f_{k,m,i\text{-}j} \leq \alpha_{i\text{-}j} (n^0_{i\text{-}j} + n_{i\text{-}j}) \overline{f}_{i\text{-}j} - \beta_t \left\{ \sigma^2_{\alpha_{i\text{-}j}} \left( (n^0_{i\text{-}j} + n_{i\text{-}j}) \overline{f}_{i\text{-}j} \right)^2 \right\}^{\frac{1}{2}}, \forall \; k, m, i\text{-}j, \tag{3.74}$$

$$-f_{k,m,i\text{-}j} \leq \alpha_{i\text{-}j} (n^0_{i\text{-}j} + n_{i\text{-}j}) \overline{f}_{i\text{-}j} - \beta_t \left\{ \sigma^2_{\alpha_{i\text{-}j}} \left( (n^0_{i\text{-}j} + n_{i\text{-}j}) \overline{f}_{i\text{-}j} \right)^2 \right\}^{\frac{1}{2}}, \forall \; k, m, i\text{-}j. \tag{3.75}$$

Note that, if the variances of $\alpha_{nt}$ and $\alpha_{i\text{-}j}$ are known, Equations (3.72)–(3.75) can be linearized. Therefore, the linearized equations are

$$\sum_m (g_{k,m,nt,p}) - \alpha_{nt} x_{nt} n_{nt,p} + \beta_g \sigma_{\alpha_{nt}} x_{nt} n_{nt,p} \leq 0, \; \forall \; k, nt, p, \tag{3.76}$$

$$\sum_m (g_{k,m,et,h}) - \alpha_{et} y_{et,h} + \beta_g \sigma_{\alpha_{et}} y_{et,h} \leq 0, \; \forall \; k, et, h, \tag{3.77}$$

$$f_{k,m,i\text{-}j} \leq \alpha_{i\text{-}j} (n^0_{i\text{-}j} + n_{i\text{-}j}) \overline{f}_{i\text{-}j} - \beta_t \sigma_{\alpha_{i\text{-}j}} (n^0_{i\text{-}j} + n_{i\text{-}j}) \overline{f}_{i\text{-}j}, \forall \; k, m, i\text{-}j, \tag{3.78}$$

$$-f_{k,m,i\text{-}j} \leq \alpha_{i\text{-}j} (n^0_{i\text{-}j} + n_{i\text{-}j}) \overline{f}_{i\text{-}j} - \beta_t \sigma_{\alpha_{i\text{-}j}} (n^0_{i\text{-}j} + n_{i\text{-}j}) \overline{f}_{i\text{-}j}, \forall \; k, m, i\text{-}j. \tag{3.79}$$

### A Six-Node System

Consider the two-stage stochastic model described by Equations (3.71), (3.76)–(3.79), (3.60), (3.61), (2.46), (2.54), and (3.64)–(3.68). Assume that the availability factor for each generating plant and the transmission capacity factor for each transmission line are standard normally distributed random variables.

The fist section of Table 3.26 shows some results for different values of $\theta_r$ when both $\beta_t$ and $\beta_g$ are fixed to zero. From the first section of Table 3.26, just as expected, it can be seen that as $\theta_r$ increases, the standard deviation decreases, and the overall cost increases.

Section two of Table 3.26 shows some results when $\theta_r = 0.03$, $\beta_t = 0$, and $\beta_g$ is allowed to vary. Section three of Table 3.26 shows the results when $\theta_r = 0.03$, $\beta_g = 0$, and $\beta_t$ varies. It can be seen that as the percentage of satisfying the probabilistic constraints increases, the overall cost also increases. Finally, the last section of Table 3.26 shows the results obtained when $\beta_g$ and $\beta_t$ are both fixed to a value of 1.3 (approximately 90% probability the chance constraints are met) and $\theta_r$ varies. Here it also can be observed the fact that a lower standard deviation implies a higher overall cost.

### A 21-Node System

As in the case of the six-node system, Table 3.27 shows different results when the parameters $\theta_r$, $\beta_t$, and $\beta_g$ take different values. It can be seen that as $\theta_r$ increases, the standard deviation decreases, and the overall cost increases. It can also be seen that as the percentage of satisfying the probabilistic constraints increases, the overall cost also increases.

Table 3.26: Annualized generating cost, gen. & trans. exp., six-node system.

| Parameter | Mean† | Std. Deviation† | Overall Cost‡ |
|---|---|---|---|
| $\theta_r = 0.00$ | 183.1240 | 39.3755 | 206.6740 |
| $\theta_r = 0.01$ | 190.5270 | 34.3488 | 208.3270 |
| $\theta_r = 0.02$ | 199.3320 | 24.2460 | 217.1320 |
| $\theta_r = 0.03$ | 184.1350 | 19.3914 | 213.8650 |
| $\theta_r = 0.04$ | 187.5800 | 16.6919 | 220.3100 |
| $\theta_r = 0.10$ | 201.7360 | 7.6376 | 231.4660 |
| $\theta_r = 0.03$, $\beta_t = 0$ | | | |
| $\approx 76\%; \beta_g = 0.7$ | 139.5090 | 19.6373 | 233.8590 |
| $\approx 84\%; \beta_g = 1.0$ | 147.1710 | 21.0679 | 241.5210 |
| $\approx 90\%; \beta_g = 1.3$ | 156.8510 | 22.9015 | 256.9510 |
| $\approx 96\%; \beta_g = 1.74$ | 223.9240 | 22.7380 | 278.1340 |
| $\theta_r = 0.03$, $\beta_g = 0$ | | | |
| $\approx 76\%; \beta_t = 0.7$ | 132.4920 | 17.1334 | 236.0920 |
| $\approx 84\%; \beta_t = 1.0$ | 134.2880 | 16.8008 | 235.6380 |
| $\approx 90\%; \beta_t = 1.3$ | 125.7790 | 16.0178 | 235.6290 |
| $\approx 96\%; \beta_t = 1.74$ | 131.1000 | 16.4021 | 236.9500 |
| $\beta_g = 1.3$, $\beta_t = 1.3$ | | | |
| $\theta_r = 0.00$ | 146.8630 | 33.1902 | 247.5130 |
| $\theta_r = 0.01$ | 148.0720 | 33.4241 | 246.4720 |
| $\theta_r = 0.02$ | 154.9880 | 15.6862 | 253.3880 |
| $\theta_r = 0.03$ | 158.5590 | 22.6046 | 256.9590 |
| $\theta_r = 0.04$ | 163.8580 | 19.0941 | 262.2580 |
| $\theta_r = 0.10$ | 181.3580 | 7.6377 | 279.7580 |

† in \$M-y; ‡ Investment cost plus annualized generating cost, in \$M-y.

Table 3.27: Annualized generating cost, gen. & trans. exp., 21-node system.

| Parameter | Mean† | Std. Deviation† | Overall Cost‡ |
|---|---|---|---|
| $\theta_r = 0.00$ | 506.4300 | 93.7604 | 679.8000 |
| $\theta_r = 0.01$ | 546.9090 | 64.6354 | 701.3790 |
| $\theta_r = 0.02$ | 490.9730 | 38.1881 | 751.8830 |
| $\theta_r = 0.03$ | 510.4170 | 25.4588 | 771.3270 |
| $\theta_r = 0.04$ | 520.1390 | 19.0941 | 781.0490 |
| $\theta_r = 0.10$ | 537.6390 | 7.6376 | 798.5490 |
| $\theta_r = 0.03$, $\beta_t = 0$ | | | |
| $\approx 76\%; \beta_g = 0.7$ | 562.6850 | 25.4588 | 858.1550 |
| $\approx 84\%; \beta_g = 1.0$ | 563.6500 | 25.4588 | 943.2700 |
| $\approx 90\%; \beta_g = 1.3$ | 614.8560 | 25.4588 | 947.1660 |
| $\approx 96\%; \beta_g = 1.74$ | 706.1340 | 25.4588 | 1027.1940 |
| $\theta_r = 0.03$, $\beta_g = 0$ | | | |
| $\approx 76\%; \beta_t = 0.7$ | 585.5910 | 25.4588 | 819.8610 |
| $\approx 84\%; \beta_t = 1.0$ | 554.6780 | 25.4588 | 820.3580 |
| $\approx 90\%; \beta_t = 1.3$ | 560.0070 | 25.4588 | 837.8970 |
| $\approx 96\%; \beta_t = 1.74$ | 530.1170 | 25.4588 | 839.0570 |
| $\beta_g = 1.3$, $\beta_t = 1.3$ | | | |
| $\theta_r = 0.00$ | 593.7580 | 112.1040 | 875.3080 |
| $\theta_r = 0.01$ | 548.2870 | 66.9735 | 911.6770 |
| $\theta_r = 0.02$ | 592.0230 | 38.5400 | 955.4130 |
| $\theta_r = 0.03$ | 611.4670 | 25.4588 | 974.8570 |
| $\theta_r = 0.04$ | 621.1890 | 19.0941 | 948.5790 |
| $\theta_r = 0.10$ | 638.6890 | 7.6376 | 1002.0790 |

† in \$M-y; ‡ Investment cost plus annualized generating cost, in \$M-y.

## 3.7 Concluding Remarks

Main contributions of this chapter are the three stochastic models for generation, transmission, and generation and transmission expansion for the vertically integrated industry. The most complete model is a two-stage mixed-integer nonlinear program that considers randomness in the demand, equivalent availability factors, and transmission capacity factors; the latter are incorporated to the model by means of probabilistic constraints. The mean-variance Markowitz theory is successfully applied as a risk minimization technique. It is shown, through various numerical examples, that as the risk aversion increases (large values of $\theta_r$) the overall cost increases and the variance of the annualized generation cost decreases; consequently, the risk is minimized. Also, as the probability of satisfying the chance constraints increases, the overall cost increases. From the EVPI obtained from the numerical examples, it is concluded that the stochastic models presented are a good representation of the uncertainties faced by the investment plans since the SS is not far from the WSS. An alternative to compute the VSS when the EEV becomes infeasible is presented. By comparing the deterministic models with the stochastic models, this chapter shows that even simple models can lead to significant savings and can be effectively applied as a first step when planning a system expansion to narrow the wide range of possibilities to just a few.

# Chapter 4

# Risk Minimization in Pool Electricity Markets

In the idealized restructured electricity industry, market participants always find the economic incentives to invest in new generation and in new transmission. In the real-world this is not the case. The opening of this chapter, presents the state-of-the-art of generation and transmission expansion planning in the re-regulated electricity industry. One of the biggest challenges that the re-regulated electricity industry faces, is the high levels of uncertainty in the supply and demand bids submitted to the ISO. Pool electricity markets, as of today, are cleared under the strong assumption of having a perfectly known future. In reality, the inability to predict what the supply and demand functions would be introduces risk into the market clearing process. Therefore, to shield against that type of risk in order to obtain a "closer-to-real-life" solution is an interesting problem. This chapter considers random variations on the levels and on the slopes of the supply and demand functions. Positive and negative correlations are considered between the corresponding coefficients of the supply and demand curves. By means of the mean-variance Markowitz theory, the risk introduced by these random variations is minimized. A comprehensive analysis of the effects that the risk minimization has on the nodal spot prices, and on the point-elasticities of the supply and demand curves is made. The non-linear optimization model presented in this chapter is validated through a three-, a six-, and a 21-node system.[1]

[1] A 57- and a 118-node system are presented in Appendix D.

This chapter is organized as follows. In Section 4.1, a review of the state-of-the-art of generation and transmission expansion in the re-regulated electricity industry is presented. Section 4.2 presents some economical definitions used in a power pool electricity market and in power system planning. Section 4.3 sketches the general problem when there is randomness in supply and demand functions; the expected social cost function is introduced. In Section 4.4, an analysis of the effects that the mean-variance Markowitz theory as a risk minimization technique has on the nodal prices and on the price-point elasticities of the supply and demand functions is given. Section 4.5 presents the general formulation of the risk minimization problem from supply and demand uncertainty for a pool electricity market. An analysis of the dual variables is done and the effects on the nodal prices and price-point elasticities is studied in a three-, a six-, and a 21-node system. Finally, Section 4.6 concludes.

## 4.1 A Few Words on Deregulation

Since the appearance of the first decentralized power systems, all the effort has been focused on the short-term period of power system economics and on how to encourage competition among market participants. When the consequences of ignoring power system planning began to surface, the attention of market participants as well as of regulators turned to power system planning. The concept of centralized system planning no longer exists in an open access scheme.

Under a power-pool market-based operation, the concept of market-driven power system expansion involves allowing market participants, without any regulatory intervention, to invest in new generation, transmission, or a combination of both as a response to economical signals such as spatial variations on LMPs. In a linearized power-flow model and ignoring losses, spatial variations on LMPs indicate that the system is congested. The presence of congestion does not necessarily mean that the reliability of the system is compromised; it means that the power system cannot support the trading patterns that the market participants are willing to make, *i.e.*, consumers at load pocket nodes are prevented, by system constraints, from importing cheap energy from producers at generation pocket nodes. There is still no general consensus as for which economical signal should prompt any system expansion. Market-driven generation expansion is easier to implement, and conceptually clearer, than market-driven transmission expansion.

For the decentralized generation expansion case, profit-maximizing market participants invest in new generation based on their expectation of energy prices, return on new investments, and demand growth. Therefore, generation expansion is motivated by price; the revenues, based on market prices, have to cover the capital and operating costs [11, 12]. When load faces real-time prices and is elastic enough, high energy prices indicate high levels of demand, and tight competitive supply and demand conditions. This, in turn, provides an incentive to invest in new generation [13]. Since demand is price-responsive in an electricity market, there exists a correlation between load and price and this affects the shape of the load duration curve, in fact, a *price duration curve* is sometimes proposed for planning [3]. A price duration curve plots the number of hours that the price equals or exceeds a preset price; it represents the price behavior of a power system over time.

A few problems may arise with transmission capacity expansion, especially when transitioning to deregulation. A power system that is designed to be operated by a vertically integrated industry presents a specific set of power flow patterns. When competition is introduced, these power flow patterns are altered. Market participants start to follow competitive rules, causing the power flow patterns to change more frequently and more significantly than ever before, making them to deviate from the flows that the original system was meant to support. This new situation creates new bottlenecks, bringing up system constraints that before deregulation were not restrictive and may create a heavily congested system [23].

The situation that prevails in many electricity markets around the world is that transmission expansion has not been able to keep up with the competition needed; in some cases, the incentives are either low or nonexistent. Market regulators have been prone to use the concept of regulated revenue through transmission access fees, providing no incentive to eliminate congestion. The absence of such an incentive is because the transmission owner's income is always the same and it is not dependant on the performance of the grid [24].

For the decentralized transmission expansion case there are numerous, and sometimes conflicting, proposals about what should incetivize transmission investments. Some of these are: i) spatial variations on LMPs, ii) FTRs, and iii) congestions costs. With regards to the spatial variations on LMPs, [14] and [15] state that the main objective of transmission expansion under deregulation is to encourage competition among participants. LMPs provide a good measure of the level of competitiveness in an electricity market. In an ideal perfectly-competitive electricity market, the price of energy is the same throughout the system and consumers are able to purchase energy from any producer without any limitation. Therefore, according to [14] and [15], the ultimate goal of transmission expansion is to equalize the LMPs throughout the system by totally alleviating the congestion, that is, to achieve a flat price profile. The idea of a flat price profile as a goal for transmission expansion rests in the fact that as a network becomes more and more congested, the gap between LMPs grows larger and larger. When the gap between LMPs is large enough, then the transmission system should be expanded. On the ground of FTRs, [16] suggests that long-term FTRs can be used to create incentives for transmission expansion. When a private investor makes an addition to the grid, it receives the revenues associated with the expansion via FTRs allocated by the ISO; in this way, the investor can recover its investment. In order for the private investor to recover its investment, [16] assumes a small variation in the LMPs defining the FTRs, *i.e.*, the *ex-post* transmission network remains with a certain level of congestion. The right level of expansion of the transmission grid is such that the remaining congestion revenue covers the investment of the expansion. Since FTRs are granted to the market participants that are involved in the transmission expansion to recover their investment [10], FTRs are to be rightly priced [17]. Because of the presence of economies of scale, any transmission expansion tends to be lumpy and it usually has a great effect on market prices. When the grid configuration is greatly affected by the expansion, there is the possibility that the prices after the investment cannot longer support it. Therefore, although the investment may be lumpy, it should be small enough so it does not have a major impact on the market prices. Depending on the size of the system, transmission investments can be made in modules, small enough to have a minimal effect on the market prices after the expansion. Another undesirable effect of lumpiness is that it can cause and under-investment in the transmission network. In any case, the project chosen must never mitigate completely the congestion nor have a large effect on LMPs. As stated above, after a transmission expansion is made, the fixed charges needed to recover the investment may be greater than the *ex-post* congestion rents awarded to the investors. It has been suggested that a combination of access fees and long-term contracts could recover the investment. Rightly priced FTRs, as a long-term guarantee, can be used to prevent the imposition of access fees in order to pay for the transmission expansion. Concerning congestion costs, [18] – [20] establish that the total congestion cost is a good criterion to measure the level of expansion needed in a transmission network. In fact, any system expansion, in generation or transmission, brings about a reduction in the congestion costs [48]. The reduction in the congestion cost is a good way of measuring the social value of the investment of any system expansion. The merchant investor must be granted the reduction in the congestion cost (total surplus created by the expansion [13]) in order to create proper incentives for the investments.[2] There should be a compromise between the congestion cost saving (social value of investment) and the transmission expansion investment cost; this will define an acceptable level of congestion after the expansion

A sound new proposition is to develop a *coordinated generation and transmission expansion plan*. The concept of having a coordinated generation and transmission expansion plan in a open access scheme, stems from the fact that generation and transmission investments can be treated as competitors. The condition of an import constrained area can be alleviated either by reinforcing the transmission link that connects the import constrained area to the system, or by building cheap generation at the import constrained area. Which investment plan is favored depends on which one of them has a greater social value.[3] An efficient power system expansion plan under an open access scheme should encourage coordination between generation and transmission expansion due to their intrinsic interrelationship. Therefore, some regulation is still needed not only to achieve this goal but also to oversee the transmission sector since, by nature, it should remain as a monopoly. Certain measure of regulation is also needed because an economic transmission investment made by market participants can affect the reliability of the system. Hence, the proposal of any transmission expansion should be first submitted to the regulator agent

[2]This is under the assumption that the supply curves represent the true marginal costs and that the demand curves represent the true willingness to pay, *i.e.*, perfect competition.

[3]For a detailed discussion on the social value of an investment whether in generation or in transmission, and on how generation and transmission expansion projects are not interchangeable, please refer to Appendix B

for approval [17]. The goal of any regulatory mechanism is to reconcile the profit-driven merchant investor's goals with the reliability-oriented planner's goals, and to reconcile the private and the public interests [10, 13, 17, 49].

## 4.2 Economic Implications of Generation and Transmission Expansion

Basic economic theory states that, under perfect competition and ignoring transmission constraints and losses, the quantity and the price for energy are determined by the intersection of the supply and demand curves. This situation can be thought as two nodes connected through a transmission line with capacity $K$ large enough so it is not binding. One node is considered as a net supply,[4] for having cheaper generation than the other, while the other is considered as a net demand.[5] The intersection of the supply and demand curves as well as the consumer and producer surpluses are shown in Figure 4.1.

When the transmission capacity $K$ limits the flow of power from the net supply node to the net demand node, expensive generation from the net demand node has to be used to satisfy the demand because of the inability to import cheap energy from the net supply node; the cost of running more expensive generation from the net supply node is known as *congestion cost* or *redispatch cost* [13]. Another way of understanding the congestion cost is as the loss of social benefit computed as the difference between net benefits without considering transmission capacity limits and considering them [19]. This out-of-merit dispatch originates two market clearing prices, one for each node, $\rho_S$ and $\rho_D$. When $K$ is binding the flow of power the transmission capacity shadow price, or the congestion price, is defined as $\zeta = \rho_D - \rho_S$. *The congestion rent* is the area defined by the congestion price and the capacity of the line, that is $\zeta K$. When transmission capacity is increased, say by $\Delta K$, the congestion cost is reduced as shown in Figure 4.2. Hence, the *social value of the investment* is the reduction of the congestion cost caused by the expansion. From

[4] $\rho$ *versus* $q$; cost increases as demand increases. It is assumed that the curve reflects the true marginal cost of production.

[5] $q$ *versus* $\rho$; demand decreases as price increases. It is assumed that the curve reflects the true marginal willingness to pay.

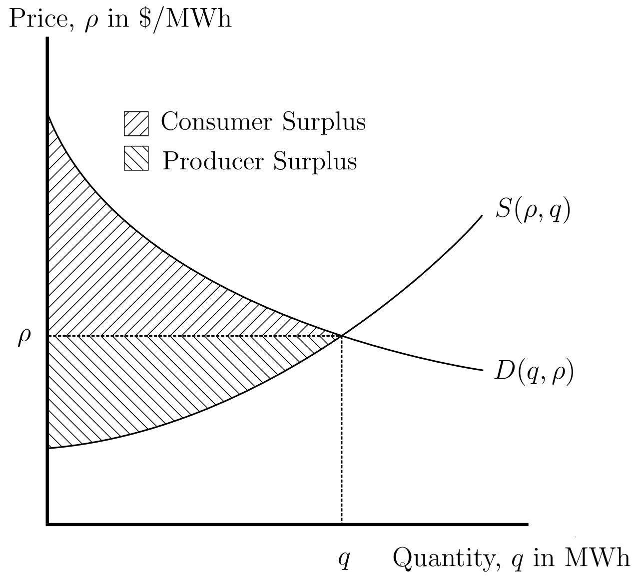


Figure 4.1: Supply and demand curves.

Figure 4.2 it can be seen that the social value of the investment can be tracked down by the changes in the producer and consumer surpluses and the change in the congestion rent after the expansion is carried out.[6] In the case of Figure 4.2, the consumer and the producer surpluses are increased while the congestion rent decreases.

If losses are to be considered, and in the presence of congestion, the nodal prices have three components: i) a component due to the cost of the marginal unit at a given swing bus, ii) a component due to losses, and iii) a component due to congestion. With nodal pricing all users pay indirectly for transmission through sale and purchase of power at the various nodes [50]. Also, the merchandizing surplus, or network revenue, is made of two parts: i) the congestion rent and ii) the cost of line losses. The merchandizing surplus can be computed as the difference in full nodal prices connected through a transmission line times the power that flows through the line. One way that the congestion rent component can be computed is by using the congestion component of the full nodal price times the power that flows through the line.

[6]Numerical examples are given in Appendix B.

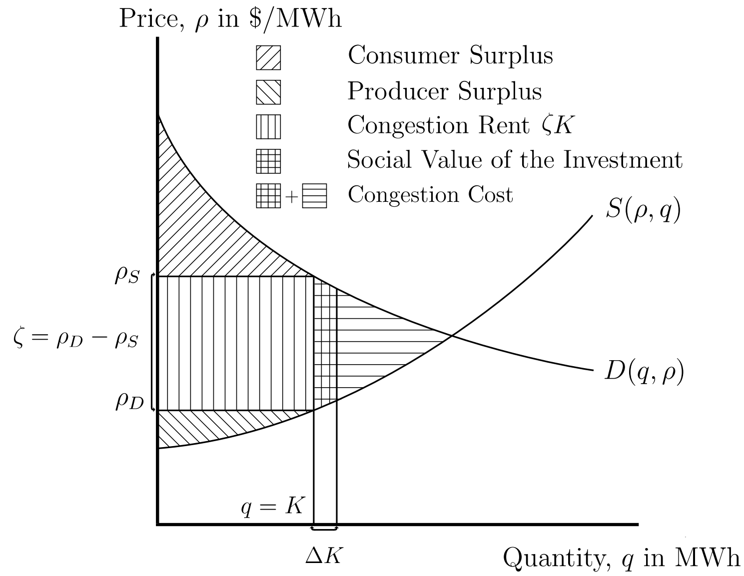


Figure 4.2: Effects of congestion.

## 4.3 The Expected Social Cost Function

The model presented in this chapter is for a double-sided pool electricity market where the ISO clears the market and also manages congestion. The ISO receives as inputs the bids of the generation and distribution companies, a double-sided pool electricity market, while considering transmission constraints. The ISO then determines the optimal levels of production and consumption, power flows, and LMPs that minimize the total social cost given by the difference between the total generation cost and the total demand benefit [48, 51].

Assume a quadratic non-decreasing concave function as the consumer's benefit and take its derivative as the affine inverse demand function.

$$B(d) = b_1 d - \frac{1}{2} b_2 d^2, \tag{4.1}$$

$$B'(d) = MB(d) = b_1 - b_2 d, \tag{4.2}$$

where $d$ is the demand, and $b_1$ and $b_2$ are constants.

Assume also a quadratic non-decreasing convex function as the supplier's variable generation cost and take its derivative as the affine supply function.

$$C(g) = a_1 g + \frac{1}{2} a_2 g^2, \tag{4.3}$$

$$C'(g) = MC(g) = a_1 + a_2 g, \tag{4.4}$$

where $g$ is the generation, and $a_1$ and $a_2$ are constants.

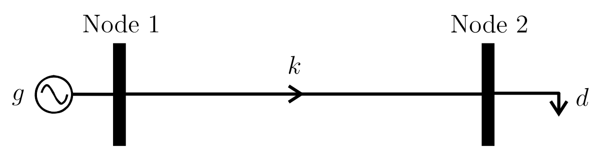


Figure 4.3: Two-node network.

The social cost function for the system shown in Figure 4.3 is

$$SC(g, d) = a_1 g + \frac{1}{2} a_2 g^2 - \left( b_1 d - \frac{1}{2} b_2 d^2 \right). \tag{4.5}$$

Under the rather strong assumption of a perfectly known future, the minimization of the social cost, Equation (4.5), renders the following closed form solution

$$g^* = d^* = \frac{b_1 - a_1}{a_2 + b_2}, \tag{4.6}$$

$$\rho^* = \frac{a_1 b_2 + a_2 b_1}{a_2 + b_2}. \tag{4.7}$$

Note that $g^*$, $d^*$, and $\rho^*$, form part of the deterministic solution to the social cost minimization problem. Unfortunately, the future cannot be perfectly predicted. A few decades ago, [1] and [52] acknowledged randomness in the levels of demand that, in turn, originate random vertical variations on the demand curve [53]. However, those results cannot be directly applied to electricity markets since they deal with linear objective functions. In this chapter, all the coefficients in Equations (4.1)–(4.4) are considered as random. Also, an analysis on the effects that the mean-variance Markowitz theory has on the elasticities of the supply and demand curves is made.

First, consider Equations (4.1) and (4.2) with the parameters $b_1$ and $b_2$ as random. Random $b_1$ accounts for vertical random variations of demand while $b_2$ accounts for random variations on the demand slope. Figures 4.4 and 4.5 show these two separate cases, respectively.

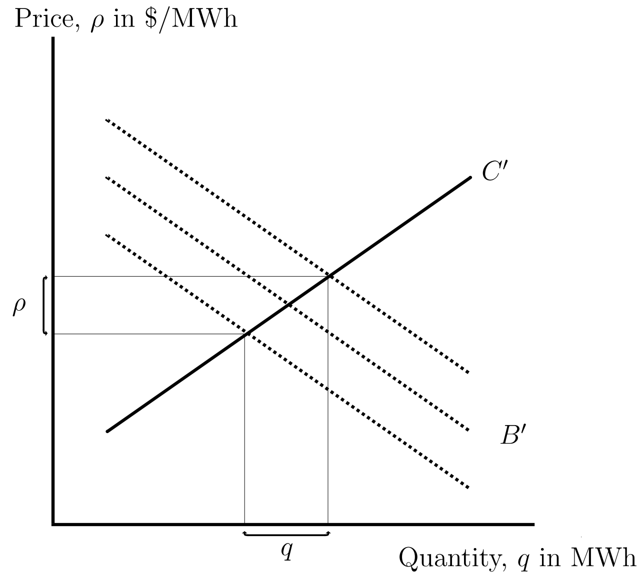


Figure 4.4: Random vertical variation of demand.

In like manner, in Equations (4.3) and (4.4), the random vertical variations of supply are represented by $a_1$ while the random variations on the supply slope are represented by $a_2$. Figures 4.6 and 4.7 show these two different cases, respectively.

Because the parameters $a_1$, $a_2$, $b_1$, and $b_2$ are considered as random, one no longer can compute a deterministic social cost function. What needs to be computed now is an expected, or average, social cost function. The expectation of any random variable gives information about its value on the average.

The expected value of the social cost when $b_1$, $a_1$, and both $b_1$ and $a_1$ are random is shown in Equations (4.8)–(4.10), respectively.

$$E\{SC\} = a_1 g + \frac{1}{2}a_2 g^2 - \left(E\{b_1\}d - \frac{1}{2}b_2 d^2\right), \tag{4.8}$$

$$E\{SC\} = E\{a_1\}g + \frac{1}{2}a_2 g^2 - \left(b_1 d - \frac{1}{2}b_2 d^2\right), \tag{4.9}$$

$$E\{SC\} = E\{a_1\}g + \frac{1}{2}a_2 g^2 - \left(E\{b_1\}d - \frac{1}{2}b_2 d^2\right). \tag{4.10}$$

The expected value of the social cost when $b_2$, $a_2$, and both $b_2$ and $a_2$ are random is

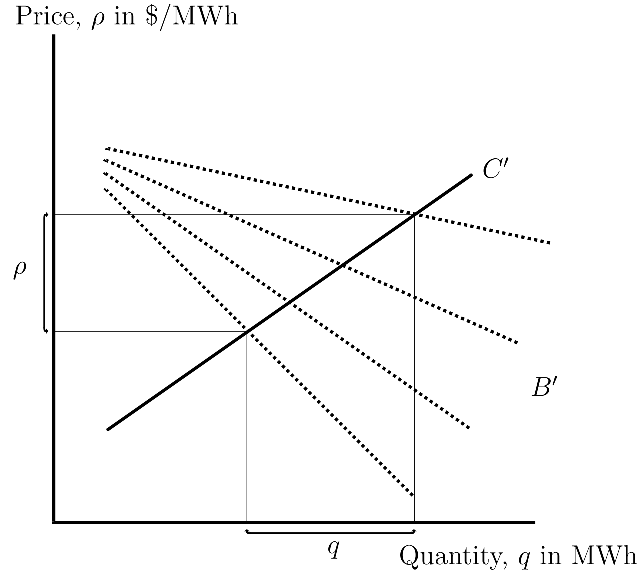


Figure 4.5: Random variation on the demand slope.

shown in Equations (4.11)–(4.13), respectively.

$$E\{SC\} = a_1 g + \frac{1}{2} a_2 g^2 - \left( b_1 d - \frac{1}{2} E\{b_2\} d^2 \right), \tag{4.11}$$

$$E\{SC\} = a_1 g + \frac{1}{2} E\{a_2\} g^2 - \left( b_1 d - \frac{1}{2} b_2 d^2 \right), \tag{4.12}$$

$$E\{SC\} = a_1 g + \frac{1}{2} E\{a_2\} g^2 - \left( b_1 d - \frac{1}{2} E\{b_2\} d^2 \right). \tag{4.13}$$

The expected social cost when $b_1$ and $b_2$; $a_1$ and $a_2$; and $b_1$, $b_2$, $a_1$, and $a_2$ are random is shown in Equations (4.14)–(4.16), respectively.

$$E\{SC\} = a_1 g + \frac{1}{2} a_2 g^2 - \left( E\{b_1\} d - \frac{1}{2} E\{b_2\} d^2 \right), \tag{4.14}$$

$$E\{SC\} = E\{a_1\} g + \frac{1}{2} E\{a_2\} g^2 - \left( b_1 d - \frac{1}{2} b_2 d^2 \right), \tag{4.15}$$

$$E\{SC\} = E\{a_1\} g + \frac{1}{2} E\{a_2\} g^2 - \left( E\{b_1\} d - \frac{1}{2} E\{b_2\} d^2 \right). \tag{4.16}$$

The inability to predict the random changes in $a_1$, $a_2$, $b_1$, and $b_2$, introduce risk in the social cost function. The goal is then to somehow minimize the risk in the expected social cost function that comes from uncertainties on the levels and on the slopes of the demand and supply functions.

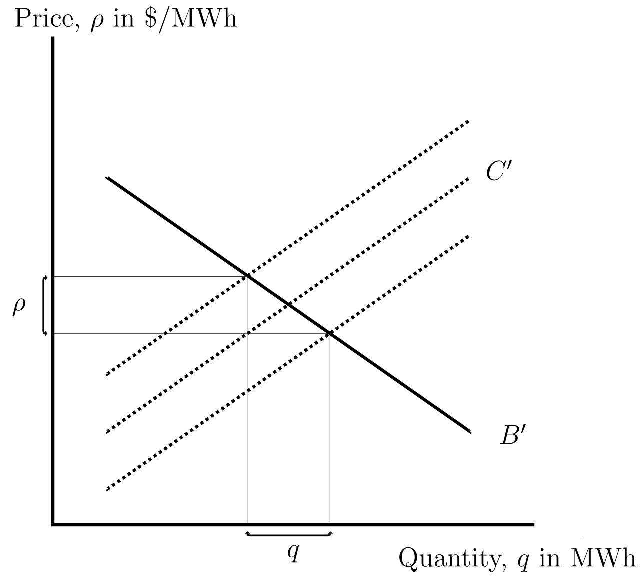


Figure 4.6: Random vertical variation of supply.

## 4.4 Mean-Variance Markowitz Theory—Impacts of Risk Minimization

Using the mean-variance Markowitz theory, the objective function that minimizes the mean value of the social cost along with its variance is

$$\min \qquad E\{SC(g,d)\} + \theta_r \sigma^2_{SC}, \tag{4.17}$$

where $\theta_r$, like before, is the risk parameter that weighs the importance that the minimization of the variance has.

### 4.4.1 Risk from supply and demand intercepts

As stated before, when there is uncertainty from the supply and demand levels, the intercepts of the supply and demand functions are considered as random variables. The variance of the social cost when $b_1$, $a_1$, and both $b_1$ and $a_1$ are random, assuming they are

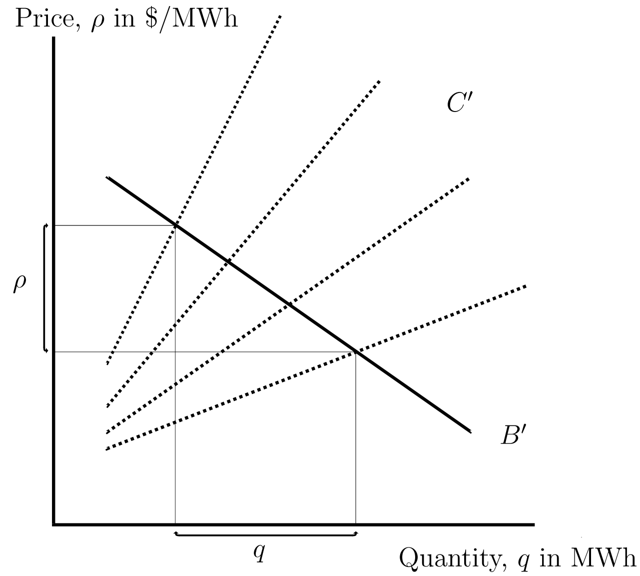


Figure 4.7: Random variation on the supply slope.

uncorrelated, is shown in (4.18)–(4.20), respectively.

$$\sigma^2_{SC} = \sigma^2_{b_1} d^2, \tag{4.18}$$

$$\sigma^2_{SC} = \sigma^2_{a_1} g^2, \tag{4.19}$$

$$\sigma^2_{SC} = \sigma^2_{a_1} g^2 + \sigma^2_{b_1} d^2. \tag{4.20}$$

For this particular situation, it is easy to obtain a closed-form solution. When *only the demand levels are random*, the minimization of the expected social cost and its variance renders

$$g^* = d^* = \frac{E\{b_1\} - a_1}{a_2 + b_2 + 2\theta_r \sigma^2_{b_1}}, \tag{4.21}$$

$$\rho^* = \frac{a_2(E\{b_1\} - a_1)}{b_2 + 2\theta_r \sigma^2_{b_1} + a_2} + a_1. \tag{4.22}$$

From Equation (4.21), one can see that as $\theta_r$ increases, the demand $d$ decreases. From Equation (4.22), since $a_1 < E\{b_1\}$ (otherwise one would have no solution), as $\theta_r$ increases, the price $\rho$ decreases.

The effects of the mean-variance Markowitz theory as a risk minimization technique can be seen from an analysis of the first order optimality conditions with respect to $d$. The

Lagrangian for this simple problem is

$$\mathcal{L}(g, d, \rho) = a_1 g + \frac{1}{2} a_2 g^2 - E\{b_1\} d + \frac{1}{2} b_2 d^2 + \theta_r \sigma_{b_1}^2 d^2 - \rho(g - d), \tag{4.23}$$

and the first order optimality condition with respect to $d$ is

$$\rho^* = E\{b_1\} - (b_2 + 2\theta_r \sigma_{b_1}^2) d. \tag{4.24}$$

Equation (4.24) shows that the minimization of the social cost variance has the equivalent effect of changing the slope of the demand function. As $\theta_r$ increases, the slope also increases, and the demand function is less sensitive to price changes. This can be seen, using Equations (4.21), (4.22), and (4.24), from the definition of demand price-point elasticity for the current operating point, that is

$$\begin{aligned} |\eta| &= \left| \frac{dd}{d\rho} \frac{\rho}{d} \right| \\ &= \left| \frac{a_1(b_2 + 2\theta_r \sigma_{b_1}^2) + a_2 E\{b_1\}}{(a_1 - E\{b_1\})(b_2 + 2\theta_r \sigma_{b_1}^2)} \right| \\ &= \left| \frac{a_1}{a_1 - E\{b_1\}} + \frac{a_2 E\{b_1\}}{(a_1 - E\{b_1\})(b_2 + 2\theta_r \sigma_{b_1}^2)} \right|. \end{aligned} \tag{4.25}$$

From Equation (4.25), with $a_1 < E\{b_1\}$, one can see that as $\theta_r$ increases the elasticity for a given point decreases. In other words, a risk averse attitude (large values of $\theta_r$) decreases the elasticity of the demand curve for a given point.[7]

When only the *supply levels are random*, the minimization of the expected social cost and its variance renders the following closed-form solution:

$$g^* = d^* = \frac{b_1 - E\{a_1\}}{a_2 + 2\theta_r \sigma_{a_1}^2 + b_2}, \tag{4.26}$$

$$\rho^* = \frac{b_2(E\{a_1\} - b_1)}{a_2 + 2\theta_r \sigma_{a_1}^2 + b_2} + b_1. \tag{4.27}$$

From Equation (4.26), one can see that as $\theta_r$ increases, the generation $g$ decreases. From Equation (4.27), since $E\{a_1\} < b_1$, as $\theta_r$ increases the price $\rho$ also increases. Note that $\theta_r$ has the opposite effect on the price with respect to the case of random demand.

[7] For the supply/demand point-elasticity, when $|\eta| < 1$ the supply/demand is inelastic and when $|\eta| > 1$ the supply/demand is elastic [54].

The Lagrangian for this simple problem is

$$\mathcal{L}(g,d,\rho) = E\{a_1\}g + \frac{1}{2}a_2 g^2 - b_1 d + \frac{1}{2}b_2 d^2 + \theta_r \sigma_{a_1}^2 g^2 - \rho(g-d), \tag{4.28}$$

and the first order optimality condition with respect to $g$ is

$$\rho^* = E\{a_1\} + (a_2 + 2\theta_r \sigma_{a_1}^2)g. \tag{4.29}$$

Equation (4.29) shows that the minimization of the social cost variance has the equivalent effect of changing the slope of the supply function. As $\theta_r$ increases, the slope increases, and the supply function becomes less sensitive to price changes. This again can be seen, using Equations (4.26), (4.27), and (4.29), from the supply price-point elasticity for a given point, with $E\{a_1\} < b_1$, that is

$$\begin{aligned} |\eta| &= \left|\frac{dg}{d\rho}\frac{\rho}{g}\right| \\ &= \left|-\frac{b_1(a_2 + 2\theta_r\sigma_{a_1}^2) + E\{a_1\}b_2}{(E\{a_1\} - b_1)(a_2 + 2\theta_r\sigma_{a_1}^2)}\right| \\ &= \left|-\left(\frac{b_1}{E\{a_1\} - b_1} + \frac{E\{a_1\}b_2}{(E\{a_1\} - b_1)(a_2 + 2\theta_r\sigma_{a_1}^2)}\right)\right|. \end{aligned} \tag{4.30}$$

A risk averse attitude decreases the elasticity of the supply curve for a given point.

Finally, when *both demand and supply levels are random*, and assuming they are uncorrelated, the minimization of the expected social cost and its variance renders the following closed-form solution:

$$g^* = d^* = \frac{E\{b_1\} - E\{a_1\}}{a_2 + b_2 + 2\theta_r(\sigma_{a_1}^2 + \sigma_{b_1}^2)}, \tag{4.31}$$

$$\rho^* = \frac{a_2 + 2\theta_r\sigma_{a_1}^2}{a_2 + b_2 + 2\theta_r(\sigma_{a_1}^2 + \sigma_{b_1}^2)}(E\{b_1\} - E\{a_1\}) + E\{a_1\}. \tag{4.32}$$

From Equation (4.31), one can see that as $\theta_r$ increases, the demand $d$/generation $g$ decreases.

The Lagrangian function is

$$\mathcal{L}(g,d,\rho) = E\{a_1\}g + \frac{1}{2}a_2 g^2 - E\{b_1\}d + \frac{1}{2}b_2 d^2 + \theta_r(\sigma_{a_1}^2 g^2 + \sigma_{b_1}^2 d^2) - \rho(g-d). \tag{4.33}$$

The first order optimality conditions with respect to $d$ and $g$ are as in Equations (4.24) and (4.29), respectively. Just like before, the minimization of the social cost variance has the equivalent effect of changing the slopes of the demand and supply functions which, in turn, affect their elasticities. As $\theta_r$ increases, both slopes increase, and the demand and supply functions become less sensitive to price changes. A risk averse attitude decreases the elasticity of both the demand and supply curves at a given point.

### 4.4.2 Risk from supply and demand slopes

A more challenging form of uncertainty, mathematically speaking, comes from random variations in the slopes of the supply and demand functions. The variance of the social cost when $b_2$, $a_2$, and both $b_2$ and $a_2$ are random, respectively, assuming they are uncorrelated, is shown in Equations (4.34)–(4.36).

$$\begin{aligned}\sigma_{SC}^2 &= \frac{1}{4}\sigma_{b_2}^2 d^4, &(4.34)\\ \sigma_{SC}^2 &= \frac{1}{4}\sigma_{a2}^2 g^4, &(4.35)\\ \sigma_{SC}^2 &= \frac{1}{4}\left(\sigma_{a2}^2 g^4 + \sigma_{b_2}^2 d^4\right). &(4.36)\end{aligned}$$

For this particular situation, it is rather difficult to obtain a closed-form solution for the minimization of the expected social cost and its variance since the objective function is a quartic equation. However, from the Lagrangian function, one can see the effects that the minimization of the variance has on the supply and demand functions. For the case when *only* $b_2$ *is a random variable*, the lagrangian function and the first order optimality condition with respect to $d$ are

$$\begin{aligned}\mathcal{L}(g,d,\rho) &= a_1 g + \frac{1}{2}a_2 g^2 - b_1 d + \frac{1}{2}E\{b_2\}d^2 + \frac{1}{4}\theta_r\sigma_{b_2}^2 d^4 - \rho(g-d), &(4.37)\\ \rho^* &= b_1 - E\{b_2\}d - \theta_r\sigma_{b_2}^2 d^3. &(4.38)\end{aligned}$$

From Equation (4.38), one can see that the minimization of the social cost variance has an equivalent effect of reshaping the demand function. It transforms the demand curve from a linear to cubic function. The more important the minimization of the variance, the larger the effect the cubic term has on the demand function. The demand price-point elasticity for a given point is

$$|\eta| = \left|\frac{E\{b_2\}d + \theta_r\sigma_{b_2}^2 d^3 - b_1}{E\{b_2\}d + 3\theta_r\sigma_{b_2}^2 d^3}\right|. \tag{4.39}$$

For the case when *only $a_2$ is a random variable*, the Lagrangian function and the first order optimality condition with respect to $g$ are

$$\mathcal{L}(g,d,\rho) = a_1 g + \frac{1}{2}E\{a_2\}g^2 - b_1 d + \frac{1}{2}b_2 d^2 + \frac{1}{4}\theta_r \sigma_{a_2}^2 g^4 - \rho(g-d), \tag{4.40}$$

$$\rho^* = a_1 + E\{a_2\}g + \theta_r \sigma_{a_2}^2 g^3. \tag{4.41}$$

It can bee seen from Equation (4.41), just like before, that the minimization of the social cost variance reshapes the supply function transforming it from a linear to a cubic function. The more important the minimization of the variance, the larger the effect that the cubic term has on the supply function. The supply price-point elasticity for a given point is

$$|\eta| = \left| \frac{E\{a_2\}g + \theta_r \sigma_{a_2}^2 g^3 + a_1}{E\{a_2\}g + 3\theta_r \sigma_{a_2}^2 g^3} \right|. \tag{4.42}$$

When *both slopes, $b_2$ and $a_2$, are considered as random*, the effect on the supply and demand functions is the same: the supply and demand function are reshaped and transformed from linear to cubic functions, *i.e.*, Equations (4.38) and (4.41). The price-point elasticity for a given point is like in Equations (4.39) and (4.42).

### 4.4.3 Risk from supply and demand intercepts and slopes

When considering randomness in the intercepts and slopes of the supply and demand curves, it is logical to assume that there exists a correlation between $b_1$ and $b_2$, and between $a_1$ and $a_2$. The covariance is a measure of how much two random variables vary together. The covariance is defined as $\sigma_{X,Y} = E\{XY\} - E\{X\}E\{Y\}$. If $\sigma_{X,Y} > 0$, $Y$ increases as $X$ increases. If $\sigma_{X,Y} < 0$, $Y$ decreases as $X$ increases.

The variance of the social cost when the intercept and the slope of the demand function, the supply function, and both the supply and demand functions are taken as random, is shown in Equations (4.43)–(4.45), respectively.

$$\sigma_{SC}^2 = \frac{1}{4}\sigma_{b_2}^2 d^4 - \sigma_{b_1,b_2} d^3 + \sigma_{b_1}^2 d^2, \tag{4.43}$$

$$\sigma_{SC}^2 = \frac{1}{4}\sigma_{a_2}^2 g^4 + \sigma_{a_1,a_2} g^3 + \sigma_{a_1}^2 g^2, \tag{4.44}$$

$$\sigma_{SC}^2 = \frac{1}{4}\left(\sigma_{a_2}^2 g^4 + \sigma_{b_2}^2 d^4\right) + \sigma_{a_1,a_2} g^3 - \sigma_{b_1,b_2} d^3 + \sigma_{a_1}^2 g^2 + \sigma_{b_1}^2 d^2. \tag{4.45}$$

Again, it is difficult to obtain a closed-form solution of the minimization of the expected social cost and its variance. The Lagrangian function, and its first order optimality condition with respect to $d$ when *only the intercept and the slope of the demand function are considered as random* are

$$
\begin{aligned}
\mathcal{L}(g,d,\rho) &= a_1 g + \frac{1}{2}a_2 g^2 - E\{b_1\}d + \frac{1}{2}E\{b_2\}d^2 \\
&\quad + \theta_r\left(\frac{1}{4}\sigma_{b_2}^2 d^4 - \sigma_{b_1,b_2}d^3 + \sigma_{b_1}^2 d^2\right) - \rho(g-d), \qquad (4.46)\\
\rho^* &= E\{b_1\} - E\{b_2\}d - \theta_r\left(\sigma_{b_2}^2 d^3 - 3\sigma_{b_1,b_2}d^2 + 2\sigma_{b_1}^2 d\right). \qquad (4.47)
\end{aligned}
$$

Minimizing the social cost variance, from Equation (4.47), reshapes the demand function; it goes from a linear function to a cubic function. The more important the risk minimization, the bigger the impact the cubic and the quadratic terms have. The demand price-point elasticity for a given point is

$$
|\eta| = \left|\frac{(E\{b_2\} + 2\theta_r\sigma_{b_1}^2)d + \theta_r(\sigma_{b_2}^2 d^3 - 3\sigma_{b_1,b_2}d^2) - E\{b_1\}}{(E\{b_2\} + 2\theta_r\sigma_{b_1}^2)d + 3\theta_r(\sigma_{b_2}^2 d^3 - 2\sigma_{b_1,b_2}d^2)}\right|. \qquad (4.48)
$$

When *only the intercept and the slope of the supply function are considered as random*, the lagrangian function and its first order optimality condition with respect to $g$ are

$$
\begin{aligned}
\mathcal{L}(g,d,\rho) &= E\{a_1\}g + \frac{1}{2}E\{a_2\}g^2 - b_1 d + \frac{1}{2}b_2 d^2 \qquad (4.49)\\
&\quad + \theta_r\left(\frac{1}{4}\sigma_{a_2}^2 g^4 + \sigma_{a_1,a_2}g^3 + \sigma_{a_1}^2 g^2\right) - \rho(g-d),\\
\rho^* &= E\{a_1\} + E\{a_2\}g + \theta_r\left(\sigma_{a_2}^2 g^3 + 3\sigma_{a_1,a_2}g^2 + 2\sigma_{a_1}^2 g\right). \qquad (4.50)
\end{aligned}
$$

Similarly, the supply function, Equation (4.50), goes from a linear to a cubic function. The supply price-point elasticity for any given fixed point is

$$
|\eta| = \left|\frac{(E\{a_2\} + 2\theta_r\sigma_{a_1}^2)g + \theta_r(\sigma_{a_2}^2 g^3 + 3\sigma_{a_1,a_2}g^2) + E\{a_1\}}{(E\{a_2\} + 2\theta_r\sigma_{a_1}^2)g + 3\theta_r(\sigma_{a_2}^2 g^3 + 2\sigma_{a_1,a_2}g^2)}\right|. \qquad (4.51)
$$

When the *intercepts and the slopes of both supply and demand curves are random variables*, the effect of the risk minimization is the same: the supply and demand functions are transformed from linear to cubic functions, like in equations (4.47) and (4.50), and their elasticities are like in Equations (4.48) and (4.51).

## 4.5 General Formulation—An Analysis on Dual Variables

The preceding analysis is for a simple radial network. It helps with the basic understanding of social cost variance minimization. This section discusses the more general case for any number of nodes and any number of lines. The mean-variance social cost minimization problem, including transmission constraints, can be defined as [3, 55]

$$\min \quad f_s(g_s, d_s) + f(\boldsymbol{g^r}, \boldsymbol{d^r}) + \theta_r[v_s(g_s, d_s) + v(\boldsymbol{g^r}, \boldsymbol{d^r})] \tag{4.52}$$

$$\text{s.t.} \quad g_s - d_s + \langle \boldsymbol{e}, \boldsymbol{g^r} - \boldsymbol{d^r} \rangle = 0, \tag{4.53}$$

$$\boldsymbol{H}(\boldsymbol{g^r} - \boldsymbol{d^r}) \leq \overline{\boldsymbol{f}}, \tag{4.54}$$

where $f_s(g_s, d_s) = E\{a_{1,s}\}g_s + \frac{1}{2}E\{a_{2,s}\}g_s^2 - (E\{b_{1,s}\}d_s - \frac{1}{2}E\{b_{2,s}\}d_s^2)$ is the expected social cost function at the swing bus, $f(\boldsymbol{g^r}, \boldsymbol{d^r}) = \sum_{\nu}^{|\mathcal{N}|-1} \big(E\{a_{1,\nu}\}g_\nu + \frac{1}{2}E\{a_{2,\nu}\}g_\nu^2 - (E\{b_{1,\nu}\}d_\nu - \frac{1}{2}E\{b_{2,\nu}\}d_\nu^2)\big)$ is the mean social cost function at every node except the swing bus, $\boldsymbol{g^r}$ is the reduced generation vector with elements $g_\nu$, $\boldsymbol{d^r}$ is the reduced demand vector with elements $d_\nu$, $\mathcal{N}$ is the set of all different nodes, $v_s(g_s, d_s) = \frac{1}{4}(\sigma_{a_{2,s}}^2 g_s^4 + \sigma_{b_{2,s}}^2 d_s^4) + \sigma_{a_{1,s},a_{2,s}} g_s^3 - \sigma_{b_{1,s},b_{2,s}} d_s^3 + \sigma_{a_{1,s}}^2 g_s^2 + \sigma_{b_{1,s}}^2 d_s^2$ is the social cost variance function at the swing bus, $v(\boldsymbol{g^r}, \boldsymbol{d^r}) = \sum_{\nu}^{|\mathcal{N}|-1} \big(\frac{1}{4}(\sigma_{a_{2,\nu}}^2 g_\nu^4 + \sigma_{b_{2,\nu}}^2 d_\nu^4) + \sigma_{a_{1,\nu},a_{2,\nu}} g_\nu^3 - \sigma_{b_{1,\nu},b_{2,\nu}} d_\nu^3 + \sigma_{a_{1,\nu}}^2 g_\nu^2 + \sigma_{b_{1,\nu}}^2 d_\nu^2\big)$ is the social cost variance function at every node except the swing bus, $\theta_r$ is the risk factor, $\boldsymbol{e}$ is the unitary vector, $\boldsymbol{H}$ is the transfer admittance matrix, and $\overline{\boldsymbol{f}}$ is the maximum value for the power-flow vector with elements $f_{i\text{-}j}$.

The Lagrangian of the problem shown in Equations (4.52)–(4.54) is

$$\begin{aligned}
\mathcal{L}(g_s, d_s, \boldsymbol{g^r}, \boldsymbol{d^r}, \rho_s, \boldsymbol{\mu}) \quad &= \quad f_s(g_s, d_s) + f(\boldsymbol{g^r}, \boldsymbol{d^r}) \\
&+ \quad \theta_r[v_s(g_s, d_s) + v(\boldsymbol{g^r}, \boldsymbol{d^r})] \\
&- \quad \rho_s[g_s - d_s + \langle \boldsymbol{e}, \boldsymbol{g^r} - \boldsymbol{d^r} \rangle] \\
&- \quad \boldsymbol{\mu}^T[\boldsymbol{H}(\boldsymbol{g^r} - \boldsymbol{d^r}) - \overline{\boldsymbol{f}}],
\end{aligned} \tag{4.55}$$

$$\mu_{i\text{-}j} \leq 0, \quad \forall\ i\text{-}j \in \mathcal{I}. \tag{4.56}$$

The first-order optimality condition with respect to generation at the swing bus is

$$\rho_s = \frac{df_s(g_s, d_s)}{dg_s} + \theta_r \frac{dv_s(g_s, d_s)}{dg_s}. \tag{4.57}$$

The first-order optimality condition with respect to generation at every node except the swing bus is

$$\begin{aligned} \nabla_{\boldsymbol{g}^r} f(\boldsymbol{g}^r, \boldsymbol{d}^r) + \theta_r \nabla_{\boldsymbol{g}^r} v(\boldsymbol{g}^r, \boldsymbol{d}^r) &= \rho_s \boldsymbol{e} + \boldsymbol{H}^T \boldsymbol{\mu}, && (4.58)\\ \boldsymbol{\rho} &= \rho_s \boldsymbol{e} + \boldsymbol{\omega}. && (4.59) \end{aligned}$$

The first-order optimality condition with respect to demand at the swing bus is

$$\rho_s = -\frac{df_s(g_s, d_s)}{dd_s} - \theta_r \frac{dv_s(g_s, d_s)}{dd_s}. \tag{4.60}$$

The first-order optimality condition with respect to demand at every node except the swing bus is

$$\begin{aligned} -\nabla_{\boldsymbol{d}^r} f(\boldsymbol{g}^r, \boldsymbol{d}^r) - \theta_r \nabla_{\boldsymbol{d}^r} v(\boldsymbol{g}^r, \boldsymbol{d}^r) &= \rho_s \boldsymbol{e} + \boldsymbol{H}^T \boldsymbol{\mu}, && (4.61)\\ \boldsymbol{\rho} &= \rho_s \boldsymbol{e} + \boldsymbol{\omega}. && (4.62) \end{aligned}$$

From Equation (4.57), one can see that the price at the swing bus now has two components. The first component is the marginal cost of the generating unit at the swing bus while the second component can be interpreted as the cost, originated by the randomness in supply at the swing bus, of minimizing the social cost variance. Note that, from Equation (4.59), the price at every other node is still affected by the congestion prices; the classical concept of congestion rents remains unchanged [13, 56]. Equations (4.60) and (4.62) can be interpreted in a similar way.

The minimization of the social cost variance, assuming that $a_{1,s}$, $a_{1,\nu}$, $a_{2,s}$, $a_{2,\nu}$, $b_{1,s}$, $b_{1,\nu}$, $b_{2,s}$, and $b_{2,\nu}$ are random variables, has a direct impact on the elasticities of the marginal cost and benefit functions. The most drastic change is that it in fact changes the supply/demand function from a linear to a cubic function.

Next, the mathematical model is evaluated[8] with a three-, six-, and a 21-node system. The numerical results are discussed and further insight is provided on the elasticities of the supply and demand curves. All the pertinent data is given in Appendix C.

Without any loss of generality, it is assumed that the intercepts and slopes of the demand functions are negatively correlated while the intercepts and slopes of the supply functions are positively correlated.

[8]All models are implemented in the optimization software GAMS using the MINOS solver [42].

**A Three-Node System**

The three-node system is shown in Figure 4.8. Take Node 3 as the slack node. All the pertinent data is given in Appendix C.

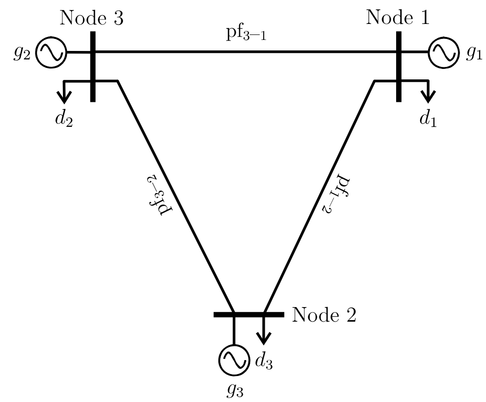


Figure 4.8: Three-node system.

Table 4.1 shows some results when the demand levels are uncertain. One can see that, as the risk aversion increases, the nodal prices, total demand, and social cost variance they all decrease. The mean social cost increases as the risk aversion increases.

Table 4.2 shows some results when the supply levels are uncertain; as $\theta_r$ increases, the nodal prices and mean social cost also increase; the demand levels, and social cost variance they all decrease.

Finally, assume that there is uncertainty from the supply and demand levels. Table 4.3 shows some results. As $\theta_r$ increases, the mean social cost also increases; the price at the swing bus (Node 3), the demand levels, and the social cost variance they all decrease.

Table 4.4 shows some results when only the slopes of the demand functions are random. It can be seen that, as $\theta_r$ increases, the social cost variance decreases. It too can be observed that the nodal prices, generation, and demand levels have an overall tendency to decrease as $\theta_r$ increases.

In Table 4.5, some results are shown for the case when the supply slopes are taken as random. As the risk aversion increases, the social cost variance decreases. Note that in this case, the nodal prices increase as $\theta_r$ increases. However, the supply and demand levels decrease as $\theta_r$ increases.

Table 4.1: Random demand levels, three-node system.

| Parameter | $\theta_r = 0$ | $\theta_r = 0.0001$ | $\theta_r = 0.001$ | $\theta_r = 0.01$ |
|---|---|---|---|---|
| $E\{SC\}^{\dagger}$ | −3479.99 | −3479.9 | −3471.95 | −3149.62 |
| ${\sigma^2_{SC}}^{\ddagger}$ | 156.58 | 154.75 | 139.93 | 70.67 |
| ${\rho_1}^{*}$ | 26.62 | 26.60 | 26.43 | 25.76 |
| ${\rho_2}^{*}$ | 33.07 | 32.99 | 32.30 | 28.14 |
| ${\rho_3}^{*}$ | 29.85 | 29.80 | 29.37 | 24.99 |
| ${g_1}^{\star}$ | 101.11 | 100.32 | 93.83 | 67.82 |
| ${g_2}^{\star}$ | 60.25 | 59.65 | 54.48 | 23.48 |
| ${g_3}^{\star}$ | 174.13 | 173.694 | 169.91 | 131.57 |
| ${d_1}^{\star}$ | 64.82 | 64.47 | 61.38 | 37.82 |
| ${d_2}^{\star}$ | 113.96 | 113.80 | 112.03 | 83.48 |
| ${d_3}^{\star}$ | 156.70 | 155.40 | 144.8 | 101.57 |

$^{\dagger}$ in \$/h, $^{\ddagger}$ $\times 10^3$ in $\$^2/\text{h}^2$, $^{*}$ in \$/MWh, and $^{\star}$ in MW.

Table 4.6 shows the case when there is randomness in the slopes of both the supply and demand curves. Just as expected, a risk averse position decreases the social cost variance. As $\theta_r$ increases, the supply and demand levels decrease. Unlike the two previous cases, the nodal prices inconsistently go up and down. One has to remember that the congestion in the transmission lines also varies as $\theta_r$ varies.

Table 4.7 gives some results when both the intercepts and slopes of the demand curves are taken as random. It can be observed that as $\theta_r$ increases, the social cost variance, nodal prices, generation, and demand levels they all decrease.

In Table 4.8, some results are presented for the case when the intercepts and slopes of the supply functions are taken as random. It can be observed that the overall tendency of the social cost variance, generation, and demand levels is to decrease as $\theta_r$ increases. Note that the nodal prices have a tendency to increase as the risk minimization increases.

Finally, Table 4.9 gives some results when the intercepts and slopes of the supply and demand functions are random variables. Notice that the nodal prices inconsistently go up and down. The tendency that prevails is that as $\theta_r$ increases, the social cost variance, generation, and demand levels they all decrease.

Table 4.2: Random supply levels, three-node system.

| Parameter | $\theta_r = 0$ | $\theta_r = 0.0001$ | $\theta_r = 0.001$ | $\theta_r = 0.01$ |
|---|---|---|---|---|
| $E\{SC\}^{\dagger}$ | −3479.99 | −3479.97 | −3478.01 | −3379.05 |
| ${\sigma_{SC}^2}^{\ddagger}$ | 63.93 | 63.49 | 59.85 | 39.19 |
| ${\rho_1}^{*}$ | 26.62 | 26.65 | 26.91 | 28.53 |
| ${\rho_2}^{*}$ | 33.07 | 33.07 | 33.12 | 33.60 |
| ${\rho_3}^{*}$ | 29.85 | 29.86 | 30.02 | 31.06 |
| ${g_1}^{\star}$ | 101.11 | 100.43 | 94.71 | 61.25 |
| ${g_2}^{\star}$ | 60.25 | 60.06 | 58.442 | 46.76 |
| ${g_3}^{\star}$ | 174.13 | 173.98 | 172.58 | 157.23 |
| ${d_1}^{\star}$ | 64.82 | 64.24 | 59.28 | 28.22 |
| ${d_2}^{\star}$ | 113.96 | 113.87 | 113.01 | 103.74 |
| ${d_3}^{\star}$ | 156.70 | 156.36 | 153.45 | 133.28 |

$^{\dagger}$ in \$/h, $^{\ddagger}$ $\times 10^3$ in \$$^2$/h$^2$, $^{*}$ in \$/MWh, and $^{\star}$ in MW.

Table 4.3: Random supply and demand levels, three-node system.

| Parameter | $\theta_r = 0$ | $\theta_r = 0.0001$ | $\theta_r = 0.001$ | $\theta_r = 0.01$ |
|---|---|---|---|---|
| $E\{SC\}^{\dagger}$ | −3479.99 | −3479.81 | −3464.77 | −2974.53 |
| ${\sigma_{SC}^2}^{\ddagger}$ | 220.51 | 216.84 | 188.45 | 78.46 |
| ${\rho_1}^{*}$ | 26.62 | 26.63 | 26.72 | 27.57 |
| ${\rho_2}^{*}$ | 33.07 | 32.99 | 32.35 | 28.75 |
| ${\rho_3}^{*}$ | 29.85 | 29.81 | 29.53 | 26.39 |
| ${g_1}^{\star}$ | 101.11 | 99.65 | 88.32 | 48.32 |
| ${g_2}^{\star}$ | 60.25 | 59.46 | 52.91 | 20.43 |
| ${g_3}^{\star}$ | 174.13 | 173.54 | 168.42 | 122.33 |
| ${d_1}^{\star}$ | 64.82 | 63.89 | 56.54 | 21.64 |
| ${d_2}^{\star}$ | 113.96 | 113.70 | 111.12 | 78.77 |
| ${d_3}^{\star}$ | 156.70 | 155.06 | 141.98 | 90.67 |

$^{\dagger}$ in \$/h, $^{\ddagger}$ $\times 10^3$ in \$$^2$/h$^2$, $^{*}$ in \$/MWh, and $^{\star}$ in MW.

Table 4.4: Random demand slopes, three-node system.

| Parameter | $\theta_r = 0$ | $\theta_r = 0.0001$ | $\theta_r = 0.001$ | $\theta_r = 0.01$ |
|---|---|---|---|---|
| $E\{SC\}^{\dagger}$ | −3480.02 | −3468.34 | −3287.16 | −2525.75 |
| ${\sigma_{SC}^2}^{\ddagger}$ | 1026.20 | 738.48 | 295.82 | 34.55 |
| ${\rho_1}^{*}$ | 26.62 | 26.47 | 26.22 | 24.81 |
| ${\rho_2}^{*}$ | 33.07 | 32.27 | 29.65 | 27.00 |
| ${\rho_3}^{*}$ | 29.85 | 29.33 | 25.76 | 22.61 |
| ${g_1}^{\star}$ | 101.11 | 95.08 | 85.51 | 31.17 |
| ${g_2}^{\star}$ | 60.25 | 54.31 | 34.72 | 14.95 |
| ${g_3}^{\star}$ | 174.13 | 169.55 | 138.29 | 110.68 |
| ${d_1}^{\star}$ | 64.82 | 65.08 | 55.51 | 39.26 |
| ${d_2}^{\star}$ | 113.96 | 114.32 | 94.73 | 55.91 |
| ${d_3}^{\star}$ | 156.70 | 139.55 | 108.29 | 61.63 |

$^{\dagger}$ in \$/h, $^{\ddagger}$ $\times\, 10^3$ in \$$^2$/h$^2$, $^{*}$ in \$/MWh, and $^{\star}$ in MW.

Table 4.5: Random supply slopes, three-node system.

| Parameter | $\theta_r = 0$ | $\theta_r = 0.0001$ | $\theta_r = 0.001$ | $\theta_r = 0.01$ |
|---|---|---|---|---|
| $E\{SC\}^{\dagger}$ | −3480.02 | −3430.99 | −3063.17 | −2278.06 |
| ${\sigma_{SC}^2}^{\ddagger}$ | 2732.60 | 1432.10 | 327.46 | 37.11 |
| ${\rho_1}^{*}$ | 26.62 | 26.99 | 28.34 | 31.17 |
| ${\rho_2}^{*}$ | 33.07 | 33.53 | 34.56 | 35.60 |
| ${\rho_3}^{*}$ | 29.85 | 30.26 | 31.45 | 33.65 |
| ${g_1}^{\star}$ | 101.11 | 103.97 | 91.24 | 60.00 |
| ${g_2}^{\star}$ | 60.25 | 61.35 | 54.84 | 35.27 |
| ${g_3}^{\star}$ | 174.13 | 146.38 | 96.97 | 53.58 |
| ${d_1}^{\star}$ | 64.82 | 57.77 | 31.78 | 0.00 |
| ${d_2}^{\star}$ | 113.96 | 105.16 | 85.39 | 65.27 |
| ${d_3}^{\star}$ | 156.70 | 148.77 | 125.89 | 83.59 |

$^{\dagger}$ in \$/h, $^{\ddagger}$ $\times\, 10^3$ in \$$^2$/h$^2$, $^{*}$ in \$/MWh, and $^{\star}$ in MW.

Table 4.6: Random supply and demand slopes, three-node system.

| Parameter | $\theta_r = 0$ | $\theta_r = 0.0001$ | $\theta_r = 0.001$ | $\theta_r = 0.01$ |
|---|---|---|---|---|
| $E\{SC\}^{\dagger}$ | −3480.02 | −3411.05 | −2914.10 | −1914.82 |
| $\sigma^2_{SC}{}^{\ddagger}$ | 3758.80 | 1919.40 | 406.72 | 39.32 |
| $\rho_1{}^{*}$ | 26.62 | 26.82 | 27.65 | 28.80 |
| $\rho_2{}^{*}$ | 33.07 | 32.81 | 31.23 | 28.80 |
| $\rho_3{}^{*}$ | 29.85 | 29.81 | 29.44 | 28.80 |
| $g_1{}^{\star}$ | 101.11 | 98.84 | 83.18 | 51.14 |
| $g_2{}^{\star}$ | 60.25 | 56.52 | 40.06 | 20.18 |
| $g_3{}^{\star}$ | 174.13 | 143.99 | 92.19 | 48.79 |
| $d_1{}^{\star}$ | 64.82 | 59.07 | 39.05 | 17.54 |
| $d_2{}^{\star}$ | 113.96 | 106.75 | 85.93 | 52.37 |
| $d_3{}^{\star}$ | 156.70 | 133.52 | 90.46 | 50.20 |

$^{\dagger}$ in \$/h, $^{\ddagger}$ $\times 10^3$ in \$$^2$/h$^2$, $^{*}$ in \$/MWh, and $^{\star}$ in MW.

Table 4.7: Random demand intercepts–slopes, three-node system.

| Parameter | $\theta_r = 0$ | $\theta_r = 0.0001$ | $\theta_r = 0.001$ | $\theta_r = 0.01$ |
|---|---|---|---|---|
| $E\{SC\}^{\dagger}$ | −3480.02 | −3451.74 | −3040.46 | −1943.96 |
| $\sigma^2_{SC}{}^{\ddagger}$ | 2285.90 | 1535.10 | 468.05 | 40.05 |
| $\rho_1{}^{*}$ | 26.62 | 26.40 | 25.87 | 23.15 |
| $\rho_2{}^{*}$ | 33.07 | 31.74 | 27.82 | 25.28 |
| $\rho_3{}^{*}$ | 29.85 | 28.65 | 23.92 | 21.02 |
| $g_1{}^{\star}$ | 101.11 | 92.47 | 72.18 | 0.00 |
| $g_2{}^{\star}$ | 60.25 | 50.29 | 21.09 | 2.13 |
| $g_3{}^{\star}$ | 174.13 | 163.61 | 122.16 | 96.70 |
| $d_1{}^{\star}$ | 64.82 | 62.47 | 45.69 | 25.68 |
| $d_2{}^{\star}$ | 113.96 | 110.30 | 79.34 | 34.29 |
| $d_3{}^{\star}$ | 156.70 | 133.61 | 90.40 | 38.85 |

$^{\dagger}$ in \$/h, $^{\ddagger}$ $\times 10^3$ in \$$^2$/h$^2$, $^{*}$ in \$/MWh, and $^{\star}$ in MW.

Table 4.8: Random supply intercepts–slopes, three-node system.

| Parameter | $\theta_r = 0$ | $\theta_r = 0.0001$ | $\theta_r = 0.001$ | $\theta_r = 0.01$ |
|---|---|---|---|---|
| $E\{SC\}^{\dagger}$ | −3480.02 | −3416.75 | −2941.71 | −1967.68 |
| $\sigma^2_{SC}{}^{\ddagger}$ | 3536.30 | 1832.30 | 413.73 | 49.12 |
| $\rho_1{}^{*}$ | 26.62 | 27.14 | 28.90 | 34.82 |
| $\rho_2{}^{*}$ | 33.07 | 33.64 | 34.93 | 35.66 |
| $\rho_3{}^{*}$ | 29.85 | 30.39 | 31.91 | 35.24 |
| $g_1{}^{\star}$ | 101.11 | 101.38 | 79.17 | 48.19 |
| $g_2{}^{\star}$ | 60.25 | 59.52 | 46.33 | 22.38 |
| $g_3{}^{\star}$ | 174.13 | 143.16 | 90.86 | 46.63 |
| $d_1{}^{\star}$ | 64.82 | 54.84 | 21.10 | 0.00 |
| $d_2{}^{\star}$ | 113.96 | 102.99 | 78.27 | 64.20 |
| $d_3{}^{\star}$ | 156.70 | 146.22 | 116.99 | 53.01 |

$^{\dagger}$ in \$/h, $^{\ddagger}$ $\times 10^3$ in \$$^2$/h$^2$, $^{*}$ in \$/MWh, and $^{\star}$ in MW.

Table 4.9: Random supply and demand intercepts–slopes, three-node system.

| Parameter | $\theta_r = 0$ | $\theta_r = 0.0001$ | $\theta_r = 0.001$ | $\theta_r = 0.01$ |
|---|---|---|---|---|
| $E\{SC\}^{\dagger}$ | −3480.02 | −3369.48 | −2646.76 | −1349.08 |
| $\sigma^2_{SC}{}^{\ddagger}$ | 5822.20 | 2782.50 | 543.67 | 40.31 |
| $\rho_1{}^{*}$ | 26.62 | 26.83 | 27.83 | 27.56 |
| $\rho_2{}^{*}$ | 33.07 | 32.30 | 29.11 | 27.56 |
| $\rho_3{}^{*}$ | 29.85 | 29.57 | 28.47 | 27.56 |
| $g_1{}^{\star}$ | 101.11 | 93.12 | 68.62 | 25.64 |
| $g_2{}^{\star}$ | 60.25 | 51.03 | 24.08 | 8.99 |
| $g_3{}^{\star}$ | 174.13 | 138.86 | 82.93 | 39.40 |
| $d_1{}^{\star}$ | 64.82 | 55.64 | 28.61 | 13.27 |
| $d_2{}^{\star}$ | 113.96 | 103.56 | 74.07 | 31.01 |
| $d_3{}^{\star}$ | 156.70 | 123.82 | 72.95 | 29.76 |

$^{\dagger}$ in \$/h, $^{\ddagger}$ $\times 10^3$ in \$$^2$/h$^2$, $^{*}$ in \$/MWh, and $^{\star}$ in MW.

### A Six-Node System

The six-node system used in this section is shown in Figure 4.9. It is a modification of the system that appears Section 3.5.2. Take Node 6 as the slack node. All the pertinent data is given in Appendix C. With this six node system, Equations (4.39), (4.42), (4.48), and (4.51) are analyzed.

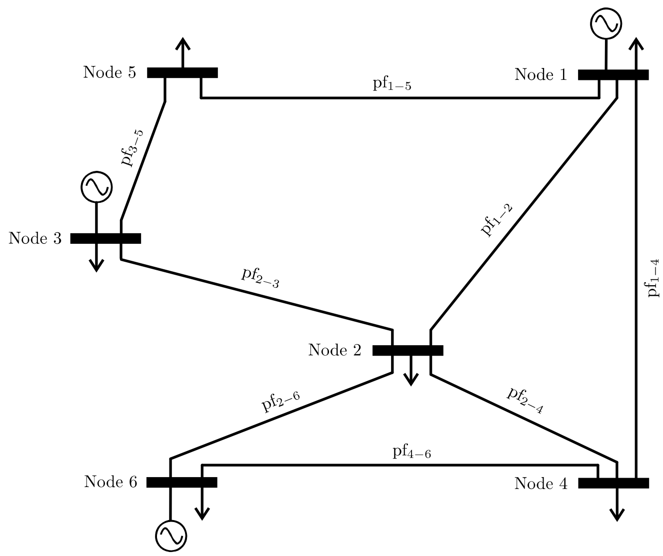


Figure 4.9: Six-node system.

Table 4.10 shows the case when the intercepts and slopes of the supply and demand curves are random variables. It can be seen that the social cost variance, supply, and demand levels decrease as the risk aversion increases. However, the nodal prices inconsistently go up and down as $\theta_r$ increases.

Take the demand located at Node 5. Using Equation (4.39), the price-point elasticity when $b_2$ is a random parameter, one can see that as $\theta_r$ varies from 0.0001 to 0.01, $|\eta|$ varies from $|-30.51|$ to $|-1.96|$. This shows then, that a risk averse position decreases the elasticity of the demand curve.

Take now the supply function of the generator located at Node 1. Using Equation (4.42), the price-point elasticity equation when $a_2$ is considered as random, one can see that as $\theta_r$ varies from 0.0001 to 0.01, $|\eta|$ varies from $|4.20|$ to $|1.16|$. This shows that a risk averse position decreases the elasticity of the supply curve at the given operating point.

Table 4.10: Random supply and demand intercepts–slopes, six-node system.

| Parameter | $\theta_r = 0$ | $\theta_r = 0.0001$ | $\theta_r = 0.001$ | $\theta_r = 0.01$ |
|---|---|---|---|---|
| $E\{SC\}^{\dagger}$ | −4410.36 | −4268.72 | −3341.10 | −1715.58 |
| ${\sigma^2_{SC}}^{\ddagger}$ | 7499.40 | 3496.10 | 695.31 | 50.77 |
| ${\rho_1}^*$ | 28.33 | 29.24 | 32.29 | 31.61 |
| ${\rho_2}^*$ | 33.08 | 32.79 | 32.29 | 31.61 |
| ${\rho_3}^*$ | 33.45 | 33.29 | 32.29 | 31.61 |
| ${\rho_4}^*$ | 31.72 | 31.77 | 32.29 | 31.61 |
| ${\rho_5}^*$ | 33.83 | 33.80 | 32.29 | 31.61 |
| ${\rho_6}^*$ | 32.40 | 32.28 | 32.29 | 31.61 |
| ${g_1}^{\star}$ | 166.67 | 149.52 | 104.90 | 39.98 |
| ${g_3}^{\star}$ | 63.10 | 57.36 | 37.35 | 16.87 |
| ${g_6}^{\star}$ | 196.52 | 152.64 | 91.67 | 43.43 |
| ${d_1}^{\star}$ | 32.04 | 14.10 | 0.00 | 0.00 |
| ${d_2}^{\star}$ | 113.81 | 97.54 | 58.96 | 24.08 |
| ${d_3}^{\star}$ | 68.14 | 63.29 | 47.67 | 20.33 |
| ${d_4}^{\star}$ | 82.19 | 71.13 | 40.83 | 18.08 |
| ${d_5}^{\star}$ | 22.48 | 22.03 | 32.81 | 15.47 |
| ${d_6}^{\star}$ | 107.62 | 91.41 | 53.65 | 22.31 |

$^{\dagger}$ in \$/h, $^{\ddagger}$ $\times 10^3$ in \$$^2$/h$^2$, $^*$ in \$/MWh, and $^{\star}$ in MW.

Take now, for instance, the demand located at Node 2. With Equation (4.48), the price-point elasticity when $b_1$ and $b_2$ are random, one can see that as $\theta_r$ varies from 0.0001 to 0.01, $|\eta|$ varies from $|-3.52|$ to $|-1.03|$. Once more, the elasticity of the demand function decreases as the risk aversion increases.

Finally, take now the supply function of the generator located at Node 1. Using Equation (4.51), the price-point elasticity when $a_1$ and $a_2$ are taken as random, one can see that as $\theta_r$ varies from 0.0001 to 0.01, $|\eta|$ varies from $|3.95|$ to $|-1.52|$. Again, one can see that as $\theta_r$ increases, the elasticity of the supply function at a given point decreases.

### A 21-Node System

The last numerical example is the 21-node system shown in Figure 4.10; it is a modification of the system in Section 3.6.2. Take Node 21 as the slack node. All the pertinent data appears in Appendix C. Table 4.11 shows how the nodal prices vary as the risk parameter varies. Just as expected, as $\theta_r$ increases the social cost variance decreases and the expected social cost increases.

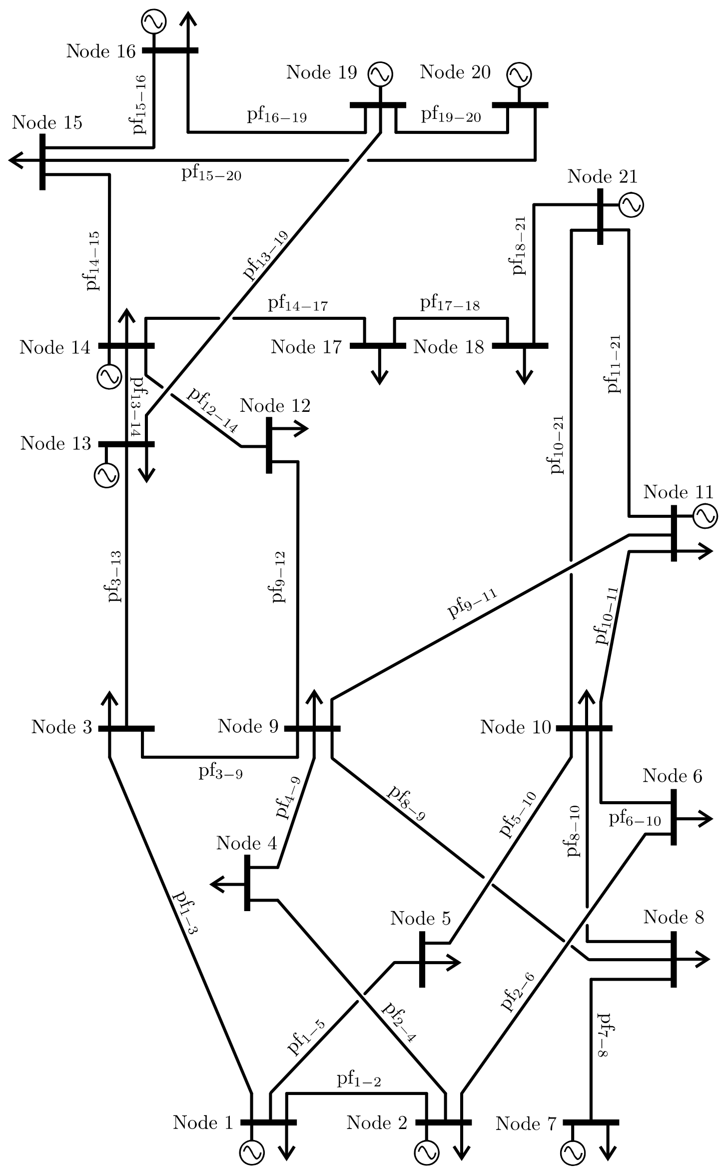


Figure 4.10: 21-one-node system.

Table 4.11: Random supply and demand intercepts–slopes, 21-node system.

| Parameter | $\theta_r = 0$ | $\theta_r = 0.0001$ | $\theta_r = 0.001$ | $\theta_r = 0.01$ |
|---|---|---|---|---|
| $E\{SC\}^{\dagger}$ | −13320.00 | −13060.00 | −10960.00 | −5977.53 |
| ${\sigma_{SC}^2}^{\ddagger}$ | 15037.00 | 8200.00 | 2043.40 | 175.45 |
| ${\rho_1}^*$ | 28.17 | 28.88 | 31.79 | 30.78 |
| ${\rho_2}^*$ | 36.00 | 35.81 | 34.57 | 30.78 |
| ${\rho_3}^*$ | 33.37 | 33.14 | 33.50 | 30.78 |
| ${\rho_4}^*$ | 34.78 | 34.51 | 34.09 | 30.78 |
| ${\rho_5}^*$ | 33.51 | 32.88 | 32.12 | 30.78 |
| ${\rho_6}^*$ | 33.85 | 33.66 | 32.97 | 30.78 |
| ${\rho_7}^*$ | 25.52 | 25.98 | 27.98 | 30.78 |
| ${\rho_8}^*$ | 33.47 | 33.22 | 33.08 | 30.78 |
| ${\rho_9}^*$ | 33.77 | 33.45 | 33.69 | 30.78 |
| ${\rho_{10}}^*$ | 33.18 | 32.99 | 32.47 | 30.78 |
| ${\rho_{11}}^*$ | 28.90 | 29.70 | 32.69 | 30.78 |
| ${\rho_{12}}^*$ | 36.62 | 36.48 | 34.64 | 30.78 |
| ${\rho_{13}}^*$ | 30.21 | 30.27 | 30.06 | 30.78 |
| ${\rho_{14}}^*$ | 31.13 | 30.94 | 30.48 | 30.78 |
| ${\rho_{15}}^*$ | 31.96 | 31.49 | 30.95 | 30.78 |
| ${\rho_{16}}^*$ | 28.09 | 28.07 | 28.45 | 30.78 |
| ${\rho_{17}}^*$ | 33.20 | 33.02 | 31.15 | 30.78 |
| ${\rho_{18}}^*$ | 36.74 | 36.59 | 32.30 | 30.78 |
| ${\rho_{19}}^*$ | 28.63 | 29.24 | 29.18 | 30.78 |
| ${\rho_{20}}^*$ | 21.39 | 22.90 | 29.87 | 30.78 |
| ${\rho_{21}}^*$ | 24.63 | 27.14 | 31.26 | 30.78 |

$^{\dagger}$ in \$/h, $^{\ddagger}$ $\times 10^3$ in $\$^2/\text{h}^2$, and $^*$ in \$/MWh

## 4.6 Concluding Remarks

An analysis of the impact that the mean-variance Markowitz theory as a risk minimization technique has on pool electricity markets is presented. Its importance is stressed due to the fact that electricity markets face increased uncertainty from random parameters in the supply and demand functions. If one is able to obtain marginal prices that consider risk minimization, better planning schemes for the deregulated electricity industry can be developed. This chapter shows that the mean-variance Markowitz theory has a direct impact on the price-point elasticities of the supply and demand curves. The most drastic change is that it transforms the linear supply and demand functions into cubic functions. The more averse the position towards risk, the lesser the elasticity of the supply and demand curves and the lesser the supply/demand levels. The nodal prices tend to vary as well as the risk parameter varies. A marginal price analysis shows that now the price at the slack node has two components: the one component is the marginal cost of the generating unit at that bus and the other is a cost component due to the social cost variance minimization. This marginal price analysis also shows that the concept of congestion prices remains unchanged. This chapter presents a fresh analysis on the economical impacts of risk minimization in power pool electricity markets.

# Chapter 5

# Conclusions

## 5.1 Summary of Contributions

The real world calls for more accurate models that take into account the effects of a variety of random events. The inability to accurately predict these random events introduce risk into the modeling process. Therefore, it is important not only to implement stochastic models but also to minimize such risk. The main contribution of this work is the successful application of the mean-variance Markowitz theory as risk minimization technique in centralized power system planning and in the market clearing process of a pool electricity market.

In Chapter 2, after addressing the goals for generation and transmission expansion in a vertically integrated industry, a joint single-stage deterministic model for generation and transmission expansion in a vertically integrated industry is proposed. The advantages of this new model are:

- Extends the single-nodal point generation planning by way of incorporating a DC model of the transmission network. This is done by assuming that there is a Load Duration Curve per node.

- Unlike the common practice of setting first a generation expansion scheme and then performing the transmission expansion planning, the joint planning for generation and transmission expansion allows the coordination between expansion projects in order to attain the common goal of minimizing the annual estimate of generation cost.

- This model incorporates important real-world constraints such as the derated capacity constraints, the stability constraint, and considers the integer nature of generation and transmission additions.

- The model indicates the location of new generation and transmission, and is able to choose from among a set of different generating technologies in order to satisfy all the operating modes considered in the Load Duration Curve.

In Chapter 3, all the deterministic models presented in Chapter 2 are modified to become two-stage stochastic models. Through various numerical examples, the superiority that stochastic models have over deterministic models is presented; important parameters to quantify randomness such as the EVPI and the VSS are also presented. The main contributions of Chapter 3 are listed below.

- The models for generation, transmission, and joint generation and transmission expansion are formulated as two-stage stochastic models. This is possible due to the structural property of block separability that some multistage stochastic programs have. Block separability allows the partitioning of the variables into aggregate-level decisions and detailed-level decisions. The generation and transmission expansion model has as aggregate-level decisions the investment in new generation and transmission, and as detailed-level decisions the annual estimate of generation.

- The mean-variance Markowitz theory is implemented in all the models as a risk minimization technique; this is done through a risk parameter that trades off variance minimization with expected value minimization of the detailed-level decisions in the objective function. Minimizing the variance is a way of minimizing the risk.

- To further include in the model the effects of random events, probabilistic constraints are incorporated. These are the probabilistic transmission line capacity constraints and the probabilistic generation derated capacity constraints.

- The application of these models, particularly the joint generation and transmission expansion model, is a first step in generation and transmission expansion planning in order to evaluate as many alternatives as possible. Once the possibilities are narrowed to just a few, more complex studies can be implemented, such as AC power flows, losses, stability, transient analysis, line design, and so forth.

- All the stochastic models for generation, transmission, and joint generation and transmission expansion that incorporate risk minimization and probabilistic constraints are a new approach to centralized power system planning. The numerical examples show that significant savings can be achieved and the expansion plan supports all the possible foreseeable scenarios.

In Chapter 4, the mean-variance Markowitz theory is applied to the market clearing process of a pool electricity market. All the coefficients of the quadratic cost and benefit functions are considered as random. The main contributions of this chapter are outlined below.

- Randomness in the nonlinear cost and benefit functions through random supply and demand curves has never been considered in the literature before. An analysis of the effects that the mean-variance Markowitz theory has, as a risk minimization technique, on the elasticities of the supply and demand functions when the intercepts and slopes are taken as random is made. It is shown that, as expected, when the risk aversion increases, the price responsiveness of both the supply and demand curves decreases.

- Through a dual variables analysis, it is shown that the classic definition of congestion rents remains unchanged. It is also shown that, at the swing bus, the nodal price has now two components. The first component is the marginal cost of the unit at the swing bus and the second component is the cost due to the minimization of the social cost variance.

- If one is able to obtain marginal prices that consider risk minimization, better planing schemes can be developed for the deregulated electricity industry.

- In an open access scheme, assuming the Independent System Operator is concerned with the social cost minimization, the social value of an investment can be used as a decision tool when comparing generation and transmission expansion projects. The social value of an investment can be tracked down by the changes in the producer and consumer surpluses, and the change in the congestion rent after an expansion is made. Therefore generation and transmission expansion projects must be treated as competitors since, in a strict sense, they are not interchangeable. Whichever plan has a greater social value of investment should be chosen.

## 5.2 Directions for Future Research

There is one major extension proposed for this work; this is to formulate a *coordinated* power system planning for the deregulated electricity industry. A coordinated power system planning can be encouraged through competition between generation and transmission expansion projects. There are several challenges to be addressed both in the formulation and the implementation. Some of these are described next.

- Power system expansion needs to take into account the investment cost minimization as well as the network operation social cost minimization. These two different objectives are in different time scales. Power system operation is in the hourly time domain (short-term) while generation and transmission investment costs are in the yearly time domain (long-term). Usually, the operational problem is a double-sided pool auction which receives as input the bids of generation companies, distribution companies (consumers) and transmission constraints. The output of this operational problem is the optimal level of production and consumption, power flow patterns, and LMPs that minimize the total social cost. The overall formulation of power system expansion planning has to bring together these two different time scales [20, 57]. Other particular challenge when it comes to the problem formulation is the integer nature of transmission line/gerating plant additions.

- Solution techniques depend on the models used to implement the power system expansion problem. Planning models can be divided into heuristic, mathematical optimization, and meta-heuristic models. Mathematical optimization models are solved with numerical optimization techniques to obtain an optimal expansion of the system. Since the objective function and the constraints implemented on these models are complex and usually non-linear, their solution is difficult and requires a considerable computational effort. Numerical optimization techniques used to solve these models are linear programming, dynamic programming, nonlinear programming, mixed integer nonlinear programming, Benders decomposition, hierarchical decomposition, interior point methods, bi-level programming, and Branch and Bound algorithms. Economic equilibrium models can also be formulated as mathematical programming problems [58] – [60]. Heuristic models use logical or empirical rules to generate and classify the options during the search; usually these models can interact with the user. Heuristic rules may include investment costs, operation costs, constraint violations, curtailment costs and so forth. Mathematically speaking, an heuristic model can obtain a good feasible solution (suboptimal) for the expansion plan. The success of heuristic models is that they are solved with small computational effort. Some models used are genetic algorithms, object oriented models, game theory, simulated annealing, expert systems, fuzzy set theory, greedy randomized adaptive search procedure, neural networks, and tabu search algorithms. Meta-heuristic models are a combination of mathematical optimization models and heuristic models [11,16,19,20,23,33], [61] – [76].

- Liberalized power system planning faces greater uncertainty than centralized power system planning. Unlike centralized planning, the siting and timing of new generation plants is unpredictable. Another source of uncertainty in liberalized power system planning is the suppliers and consumers bids for energy. Consequently, after developing a coordinated generation and transmission expansion model for the deregulated electricity industry, all these uncertainties need to be taken into account and risk minimization techniques should also be implemented.

# Appendix A

# Stochastic Programming

Stochastic Programming is used when optimal decisions are to be taken when not all the information is available, it is not known with certainty, or it cannot be perfectly predicted. In real life many of the constraints in an optimization problem are not know with accuracy, in fact, are dependant on environment-determining variables (random variables). Therefore, deterministic optimization methods are no longer suitable to solve these type of problems.

A classical application of stochastic programming in power systems is capacity expansion of power plants. There are many mathematical models in the literature for stochastic generation expansion [5, 28], [77] – [79].

This chapter is organized as follows: Section A.1 gives a brief introduction on random variables. Section A.2 gives a comparison between deterministic and stochastic programming. Section A.3 introduces the multistage stochastic program with fixed recourse and, using the property of block separability, it is transformed into a two-stage program. Finally, a detailed analysis of two-stage stochastic programs with fixed recurse is presented in Section A.4.

## A.1 Random Variables

A *random variable* maps each point, or outcome $\omega$, in the sample space $\Omega$ of a random (chance) experiment to a point in the real line. A random variable takes on a given numerical value with some specified probability. Therefore, a random variable can be thought as a function or rule of assignment. It is important to note that more than one point in the sample space can be mapped into the same point on the real line. If the mapping is to the $n$-dimensional Euclidean space, the mapping is called a *random vector* [80]. A random variable[1] can be written as

$$k = K(\omega), \tag{A.1}$$

where

- $\omega$ is an element in the sample space $\Omega$,
- $K$ is the random variable, and
- $k$ is the numerical value of the random variable $K$ assumed for a given element $\omega$ of the sample space $\Omega$.

Equation A.1 is depicted in Figure A.1.

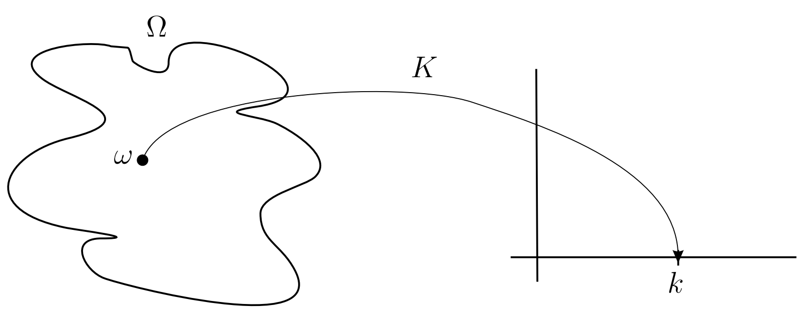


Figure A.1: Random variable $K(\omega) = k$.

Random variables can be discrete, continuous, or mixed. *Discrete* random variables take on only a countable number of possible values. The probability that the discrete random variable $K$ takes on the value $k$ can be expressed with the function $f_K[k] = \Pr[\omega \in \Omega :$

[1]It is customary in probability to denote the random variable by capital letters ($X, Y, Z$, or the like), an arbitrary point in the sample space by lowercase Greek letters ($\zeta, \omega, \nu$, or the like), and the value of the random variable assumed for a given sample space element by the corresponding lowercase letter ($x, y, z$, or the like) [81].

$K(\omega) = k]$. The function $f_K[k]$ assigns probabilities to *points* of the real line and its plot is known as the Probability Mass Function (PMF). *Continuous* random variables take on a continuum of values and it is assumed that any of these values occurs with infinitesimal probability. The probability that the continuous random variable $K$ takes on the value $k$ can be expressed with the function $F_K[k] = \Pr[\omega \in \Omega : K(\omega) \leq k]$. The function $F_K[k]$ assigns probabilities to *intervals* of the real line and its plot is known as the Probability Distribution Function (PDF). *Mixed* random variables take on a discrete set of values, each with finite probability, as well as a continuum of values, a specific value of which has infinitesimal probability [80, 81].

From probability theory [5, 46, 82], a *probability space* $(\Omega, \mathcal{F}, P)$ describes the sample space $\Omega$ and the family $\mathcal{F}$ of all possible events $F$ with an associated probability measure $P$. The sample space $\Omega$ is the set of all outcomes $\omega$ that a random experiment can have, thus $\omega \in \Omega$. The outcomes can form subsets of $\Omega$ called events and are denoted by $F$, thus $F \subset \Omega$. To each event $F \in \mathcal{F}$ is associated a value $P(F)$ called a probability. Probabilities can also be assigned to a specific outcome of a random variable. Some probability properties are the following:

- $0 \leq P(F) \leq 1$.
- $P(\emptyset) = 0$.
- $P(\Omega) = 1$.
- $P(F_1 \cup F_2) = P(F_1) + P(F_2)$ if $F_1 \cap F_2 = \emptyset$.

## A.2 Deterministic *vs.* Stochastic Programming

A deterministic optimization problem [83,84] minimizes (or maximizes) the objective function $g_0$ over the feasible set of solutions $\mathcal{S}$. The feasible set of solutions $\mathcal{S}$ can be defined as

$$\mathcal{S} = \{x \in \Re^n \mid x \in X, g_i(x) \leq 0, \ \forall \ i = 1, \dots, m\}, \tag{A.2}$$

where

- $\Re^n$ is the set of $n$-dimensional real vectors,
- $X \subset \Re^n$, and
- $g_i(x)$ is a real-valued function on $\Re^n$, $\forall\ i = 1, \ldots, m$.

The deterministic optimization problem can be stated as

$$\min \quad z = g_0(x) \tag{A.3}$$

$$\text{s.t.} \quad g_i(x) \leq 0,\ \forall\ i = 1, \ldots, m, \tag{A.4}$$

where $x \in X \subset \Re^n$.

A deterministic optimization problem has the objective function $g_0(x)$ and all the constraints $g_i(x)$ well defined; hence, the future is assumed to be perfectly known. In real life this is not possible since the future cannot be predicted with certainty; in fact, the constraints $g_i(x)$ are also dependant on the *random vector* $\tilde{\xi}$, that is, $g_i(x, \tilde{\xi})$.

The stochastic optimization problem can be stated as

$$\min \quad z(\omega) = g_0(x, \tilde{\xi}) \tag{A.5}$$

$$\text{s.t.} \quad g_i(x, \tilde{\xi}) \leq 0,\ \forall\ i = 1, \ldots, m, \tag{A.6}$$

where $x \in X \subset \Re^n$ minimizes the objective function on the corresponding set of feasible solutions, and $\tilde{\xi}$ is a random vector[2] that varies over a set $\Xi \subset \Re^q$. In this formulation, it is assumed that for every event $F \in \mathcal{F}$ and $F \subset \Xi$, the probability $P(F)$ is known and is independent of $x$. The constraints are random variables, that is, $g_i(x, \cdot) : \Xi \mapsto \Re\ \forall\ x, i$ [46, 82].

## A.3 Multistage Stochastic Programming with Fixed Recourse

Decision problems are hardly solved in two stages; they usually are solved after taking several decisions that adapt to outcomes as they evolve over time. A more suitable way to

[2] In $\Re^q$ one can only represent sets of the type $I_{[a,b)} = \{x \in \Re^q | a_i \leq x_i < b_i, i = 1, \cdots, q\}$.

model real life problems is by means of a *multistage stochastic model with fixed recourse*. The following formulation is based on [5, 78].

The multistage stochastic linear program with fixed recourse can be written as

$$\min \quad z = c^{1\prime}x^1 + E_{\zeta^2}\{\min c^2(\tilde{\xi})'x^2(\tilde{\xi}^2) + \cdots + E_{\zeta^H}\{\min c^H(\tilde{\xi})'x^H(\tilde{\xi}^H)\}\cdots\} \tag{A.7}$$

$$\text{s.t.} \quad W^1x^1 = h^1, \tag{A.8}$$

$$T^1(\tilde{\xi})x^1 + W^2x^2(\tilde{\xi}^2) = h^2(\tilde{\xi}), \tag{A.9}$$

$$\cdots\vdots$$

$$T^{H-1}(\tilde{\xi})x^{H-1}(\tilde{\xi}^{H-1}) + W^Hx^H(\tilde{\xi}^H) = h^H(\tilde{\xi}), \tag{A.10}$$

$$x^1 \geq 0, \tag{A.11}$$

$$x^t(\tilde{\xi}^t) \geq 0, \ \forall\, t = 2, \ldots, H, \tag{A.12}$$

where[3] $c^1 \in \Re^{n_1}$, and $h^1 \in \Re^{m_1}$ are known vectors; $W^1 \in \Re^{m_1\times n_1}$ is a *known* matrix (fixed recourse, not dependant on $\tilde{\xi}$). $c^t(\tilde{\xi}) \in \Re^{n_t}$, $h^t(\tilde{\xi}) \in \Re^{m_t}$, $T^{t-1}(\tilde{\xi}) \in \Re^{m_t\times n_{t-1}}$, and $W^t \in \Re^{m_t\times n_t}$ (known matrices) are for all $t = 2,\ldots,H$; $\tilde{\xi}$ is defined on $(\Xi, \mathcal{F}, P)$. $\zeta^t(\tilde{\xi})' = (c^t(\tilde{\xi})', h^t(\tilde{\xi})', T_1^{t-1}(\tilde{\xi}), \ldots, T_{m_t}^{t-1}(\tilde{\xi}))$ is a random $N_t$-vector on $(\Xi, \Sigma^t, P)$ for all $t = 2,\ldots,H$ and $\Sigma^t \subset \Sigma^{t+1}$. Note that the dependance of $x^t$ on $\tilde{\xi}^t$ is not functional as in the case of $c^t(\tilde{\xi})$, $T^{t-1}(\tilde{\xi})$, and $h^t(\tilde{\xi})$; it indicates that the decisions $x^t(\tilde{\xi}^t)$ are not the same under different realizations up to time $t$ of $\tilde{\xi}^t$. Decisions $x^t(\tilde{\xi}^t)$ are chosen so that the involved constraints hold almost surely for all $\xi^t \in \Xi$.

Using the principles of dynamic programming, and with the stages of the stochastic program going from 1 to $H$, one can define states for a deterministic dynamic program as $x^t(\tilde{\xi}^t)$. Note that the only interconnection between periods is through the realization of $x^t(\tilde{\xi}^t)$. For the last period, from Equations (A.7)–(A.12), one has

$$Q^H\left(x^{H-1}, \zeta^H(\tilde{\xi})\right) = \min \quad c^H(\tilde{\xi})'x^H(\tilde{\xi}) \tag{A.13}$$

$$\text{s.t.} \quad T^{H-1}(\tilde{\xi})x^{H-1} + W^Hx^H(\tilde{\xi}) = h^H(\tilde{\xi}), \tag{A.14}$$

$$x^H(\tilde{\xi}) \geq 0. \tag{A.15}$$

Letting $\mathcal{Q}^{t+1}(x^t) = E_{\zeta^{t+1}}\left[Q^{t+1}(x^t, \zeta^{t+1}(\tilde{\xi}))\right]$, $\forall\ t$, the dynamic programming type of re-

[3] The superscript 1 is used only to stress that it is the first stage.

coursion for $t = 2, \ldots, H-1$ is

$$Q^t\left(x^{t-1}, \zeta^t(\tilde{\xi})\right) = \min \quad c^t(\tilde{\xi})' x^t(\tilde{\xi}) + \mathcal{Q}^{t+1}(x^t) \tag{A.16}$$

$$\text{s.t.} \quad T^{t-1}(\tilde{\xi}) x^{t-1} + W^t x^t(\tilde{\xi}) = h^t(\tilde{\xi}), \tag{A.17}$$

$$x^t(\tilde{\xi}) \geq 0, \tag{A.18}$$

where $x^t$ indicates the state of the system.

### A.3.1 Block separable recourse

Multistage stochastic programs have, either naturally or by manipulation, a very particular structural property: block separability. A multistage stochastic problem is in a way a set of subproblems in a decision tree. The decision vector can be divided into an aggregate-level decision vector and a detailed-level decision vector. Because of this partition in two of the decision vector, the objective function and the constraint matrices are also partitioned in two. The following definition is taken from [5].

A multistage stochastic linear program has *block separable recourse* if for all periods $t = 1, \ldots, H$ and all $\tilde{\xi}$, the decision vectors, $x^t(\tilde{\xi})$, can be written as $x^t(\tilde{\xi}) = (w^t(\tilde{\xi}), y^t(\tilde{\xi}))$ where $w^t$ represents aggregate level decisions and $y^t$ represents detailed level decisions. The constraints also follow these partitions:

1. The stage $t$ objective contribution is $c^{t\prime} x^t(\tilde{\xi}) = r^{t\prime} w^t(\tilde{\xi}) + q^{t\prime} y^t(\tilde{\xi})$.

2. The constraint matrix $W^t$ is block diagonal:

$$W^t = \begin{pmatrix} A^t & 0 \\ 0 & B^t \end{pmatrix}. \tag{A.19}$$

3. The other components of the constraints are random but we assume that for each realization of $\tilde{\xi}$, $T^t(\tilde{\xi})$ and $h^t(\tilde{\xi})$ can be written:

$$T^t(\tilde{\xi}) = \begin{pmatrix} R^t(\tilde{\xi}) & 0 \\ S^t(\tilde{\xi}) & 0 \end{pmatrix} \text{ and } h^t(\tilde{\xi}) = \begin{pmatrix} b^t(\tilde{\xi}) \\ d^t(\tilde{\xi}) \end{pmatrix}, \tag{A.20}$$

where the zero components of $T^t$ correspond to the detailed level variables.

### A.3.2 Block separable multistage programs as two-stage programs

When a multistage stochastic program has block separable recourse, it is possible to transform it into a two-stage stochastic program. Detailed-level variables have no direct effect on future constraints. Aggregate-level variables can be grouped together and sent into the first stage. The first stage is then composed of the aggregate-level decisions while the second stage is composed of the detailed-level decisions. Because of this separation, a multistage stochastic problem can be transformed into a two-stage stochastic problem. Multistage stochastic integer programs can also be transformed into two-stage programs as long as the integer variables are associated with the aggregate level decisions. For an extensive discussion on how to transform multistage stochastic programs into two-stage stochastic programs, please refer to [78].

## A.4 Two-Stage Stochastic Programming with Fixed Recourse

The following discussion is taken in part [5, 46, 79, 85].

The two-stage recourse problem incorporates the characteristics of the *anticipative model* and the *adaptive model* of stochastic optimization. The two-stage recourse mathematical model is a trade-off between the long-term anticipatory strategies and the short-term adaptive adjustments.

### A.4.1 Anticipative model

The *anticipative optimization* model is used to plan in view of all the possible future values or realizations of $\tilde{\xi}$. To do this, the frequency of occurrence—or probabilities—of all the possible values of $\tilde{\xi}$ are used.

One probabilistic *feasibility* definition of the stochastic problem that includes the mean[4]

[4]The mean or expectation can be thought as the average value of the random variable. The average is weighed by the probabilities. In the case of a discrete random variable, the expectation is defined

of the random variable $g_i(x, \cdot)$ can be defined as some $x \in X$ that satisfies the constraints $g_i(x, \tilde{\xi}) \leq 0,\ \forall\ i = 1, \ldots, m$ with a certain level of reliability

$$\Pr\ \{\tilde{\xi} \mid g_i(x, \tilde{\xi}) \geq 0,\ \forall\ i = 1, \ldots, m\} \geq \alpha, \tag{A.21}$$

where $\alpha \in (0, 1)$ is a preestablished reliability level, or $x \in X$ can be in the mean of the random variable $g_i(x, \cdot)$, that is,

$$E\{g_i(x, \tilde{\xi})\} \leq 0,\ \forall\ i = 1, \ldots, m. \tag{A.22}$$

Another probabilistic *feasibility* definition that includes the mean and the variance[5] of the random variable $g_i(x, \cdot)$ is

$$E\{g_i(x, \tilde{\xi})\} + \beta\sqrt{\mathrm{Var}\{g_i(x, \cdot)\}} \leq 0, \tag{A.23}$$

where $\beta$ is a positive constant.

A probabilistic *optimality* definition can be expressed in terms of the feasible $x$ that minimizes

$$\Pr\ \{\tilde{\xi} \mid g_0(x, \tilde{\xi}) \geq \alpha_0\}, \tag{A.24}$$

where $\alpha_0 \in (0, 1)$ is a preestablished reliability level, or $x$ minimizes the expected value of the future objective function

$$E\{g_0(x, \tilde{\xi})\}. \tag{A.25}$$

A general formulation for stochastic optimization problems in the probability space $(\Xi, \mathcal{F}, P)$ is presented next:

$$\begin{aligned}
\min \quad z &= F_0(x) \\
&= E\{f_0(x, \tilde{\xi})\} \\
&= \int f_0(x, \tilde{\xi}) P(d\tilde{\xi}) && \text{(A.26)} \\
\text{s.t.}\ \ F_i(x) &= E\{f_i(x, \tilde{\xi})\} \\
&= \int f_i(x, \tilde{\xi}) P(d\tilde{\xi}) \leq 0,\ \forall\ i = 1, \ldots, m, && \text{(A.27)}
\end{aligned}$$

as $\mathrm{E}\{K\} = \sum_\Omega K(\omega)\Pr[\omega]$. In the case of a continuous random variable, the expectation is defined as $\mathrm{E}\{K\} = \int_\Omega K(\omega) d\mathrm{P}(\omega)$ [80].

[5] The variance is the squared value of the standard deviation. The standard deviation $\sigma$ can be thought as the average spread about the mean of the random variable. Hence, the variance is defined as $\sigma_X^2 = \mathrm{Var}(X) = E\{X^2\} - E^2\{X\}$ [81].

where

- $x \in X \subset \Re^n$,
- $f_i : \Re^n \times \Xi \rightarrow \Re, \; \forall \; i = 1, \ldots, m$,
- $f_0 : \Re^n \times \Xi \rightarrow \bar{\Re}$, and
- $\bar{\Re}$ is the set of extended real numbers [83].

### A.4.2 Adaptive model

In contrast with the anticipative optimization model where a decision $x$ is made in view of all the possible future values of $\tilde{\xi}$, the *adaptive model* makes an observation before choosing $x$.

An observation gives a partial description of the random vector $\tilde{\xi}$. Assume that $\mathcal{B} \subset \mathcal{F}$ is a collection of events that contains the information after an observation is made. The decision $x$ is a function of $\tilde{\xi}$, in turn, the values of $\tilde{\xi}$ depend on $\mathcal{B}$, this in turn, implies that the decision $x$ is a function of $\mathcal{B}$. The stochastic optimization problem for each $\tilde{\xi} \in \Xi$ can be stated as

$$\min \quad z(\tilde{\xi}) = E\{f_0(x, \cdot)|\mathcal{B}\}(\tilde{\xi}) \tag{A.28}$$

$$\text{s.t.} \quad E\{f_i(x, \cdot)|\mathcal{B}\}(\tilde{\xi}) \leq 0, \; \forall \; i = 1, \cdots, m, \tag{A.29}$$

where $x \in X \subset \Re^n$, and $E\{\cdot|\mathcal{B}\}$ is the conditional expectation given $\mathcal{B}$.

When $\mathcal{B} = \mathcal{F}$ ($\tilde{\xi}$ becomes completely known), the stochastic optimization problem for all $\tilde{\xi}$ becomes

$$\min \quad z(\tilde{\xi}) = f_0(x, \tilde{\xi}) \tag{A.30}$$

$$\text{s.t.} \quad f_i(x, \tilde{\xi}) \leq 0, \; \forall \; i = 1, \cdots, m, \tag{A.31}$$

where $x \in X \subset \Re^n$.

### A.4.3 The recourse model

As mentioned before, the recourse model brings together the properties of the anticipative model and the adaptive model. For the sake of simplicity, *the two-stage stochastic linear program with fixed recurse* is analyzed, the same concepts apply for non-linear programs.

The general definition of a deterministic linear program is

$$\begin{aligned} \min \quad & z = c'x && \text{(A.32)} \\ \text{s.t.} \quad & Ax = b, && \text{(A.33)} \\ & x \geq 0, && \text{(A.34)} \end{aligned}$$

where $c, x \in \Re^n$; $b \in \Re^m$; and $A \in \Re^{m \times n}$.

Assume that $b$ is not known with certainty but its probability distribution function is available. Under these circumstances, $x$ cannot be found to satisfy the constraint $Ax = b$ for whatever value of $b$. The discrepancy to equate $Ax$ and $b$ is a random variable that depends on $x$. Assume that a penalty function for such discrepancy is given. The objective function can be defined as the minimization of the sum between $c'x$ and the expected value of the penalty function (or recourse function). The classic formulation, developed in 1955 by Beale and Dantzig, is

$$\begin{aligned} \min \quad & z = c'x + E\{Q(x, \tilde{\xi})\} && \text{(A.35)} \\ \text{s.t.} \quad & Ax = b, && \text{(A.36)} \\ & x \geq 0, && \text{(A.37)} \end{aligned}$$

where

$$\begin{aligned} Q(x, \tilde{\xi}) = \quad \min \quad & q(\tilde{\xi})' y(\tilde{\xi}) && \text{(A.38)} \\ \text{s.t.} \quad & T(\tilde{\xi}) x + W y(\tilde{\xi}) = h(\tilde{\xi}), && \text{(A.39)} \\ & y(\tilde{\xi}) \geq 0. && \text{(A.40)} \end{aligned}$$

Equations (A.35)–(A.37) are the first stage problem while Equations (A.38)–(A.40) are the second stage problem. The first stage decisions are represented by $x \in \Re^{n_1}$. Related to $x$ are the vector $c \in \Re^{n_1}$, the vector $b \in \Re^{m_1}$, and the matrix $A \in \Re^{m_1 \times n_1}$. In the second stage, the random vector $\tilde{\xi}$ is defined in the probability space $(\Xi, \mathcal{F}, P)$. For a given realization of the random vector $\tilde{\xi}$ the second stage data for $q(\tilde{\xi}) \in \Re^{n_2}, h(\tilde{\xi}) \in \Re^{m_2}$, and $T(\tilde{\xi}) \in \Re^{m_2 \times n_1}$ become available; each one of these is also a random variable. The matrix $W \in \Re^{m_2 \times n_2}$ is known as the recourse matrix. When the recourse matrix is dependant on $\tilde{\xi}$, it is said that the problem has *random recourse*; when $W$ is not dependant on $\tilde{\xi}$, it is said that the problem has *fixed recourse*.

The set of feasible first stage decisions is

$$X = \{x \in \Re^{n_1} | Ax = b, x \geq 0\}. \tag{A.41}$$

The set of dual feasible solutions for the second stage problem is

$$\Pi = \{\pi \in \Re^{m_2} | \pi W \leq q\}. \tag{A.42}$$

The problem in Equations (A.35)–(A.40) can be restated as

$$\min \quad z = c'x + E\{Q(x, \tilde{\xi})\} \tag{A.43}$$

$$\text{s.t.} \quad Ax = b, \tag{A.44}$$

$$T(\tilde{\xi})x + Wy(\tilde{\xi}) = h(\tilde{\xi}), \tag{A.45}$$

$$x \geq 0, \tag{A.46}$$

$$y(\tilde{\xi}) \geq 0. \tag{A.47}$$

This problem, put in words is: Find the "here-and-now" solution $x$ before the components of $b$ become known; when they become known, a *recourse* $y$ must be found from the second stage problem to minimize the penalty function.

Unlike $q(\tilde{\xi})$, $h(\tilde{\xi})$, and $T(\tilde{\xi})$, the dependence of $y(\tilde{\xi})$ is not functional; it only indicates that different decisions $y$ are taken depending of different realizations of $\tilde{\xi}$. It must be noted that the expectation of the second stage objective $Q$ is taken over all realizations of the random vector $\tilde{\xi}$.

# Appendix B

# Generation *v.s.* Transmission Expansion

In the idealized restructured electricity industry the concept of centralized power system planning no longer exists. Market participants are responsible for the investment in new generation and in new transmission. In this ideal world, market participants always find economic incentives to promote a reliable and economic operation of the power system. In the real world, only generation expansion has been left to market participants with fairly acceptable results so far. Transmission expansion however, has not been able to keep up with the participant's trading patterns. To ensure a reliable and economic operation of a power system, regulatory intervention has played a key role in assessing and evaluating any proposed investment in new transmission. The ISO, in its role as system overseer, faces the multi-objective problem of evaluating and inducing efficient investments by using economical signals and issuing various types of orders. The ISO might choose as an objective the minimization of the social cost (maximization of social welfare), minimization of local market power, the maximization of consumer surplus, the minimization of congestion rents, the minimization of congestion cost, or even security measures [86]. As in the case of centralized power system planning, these different optimization criteria are inconsistent.

In general, generation and transmission expansion projects can be either a substitute or a complement of each other. The situation of an import constrained area can be alleviated either by reinforcing the transmission link that connects the import constrained area to the system, or by building cheap generation at the import constrained area. In a strict sense, generation is not an exact substitute for transmission and *vice versa*. When elastic demands are considered, upgrading the link between a generation and an import constrained area causes an increase in demand and a decrease in expensive generation at the import constrained area. This in turn causes an increase in the consumer surplus that out weighs the decrease in the producer surplus at the import constrained area. Also, it is of paramount importance to consider the effects that network upgrades have on the transfer admittance matrix since the power that flows in the system is dependant on such matrix. When modeling transmission expansion projects it is not enough to increase the transmission limits but also to upgrade the transfer admittance matrix. All the previous assertions are exemplified by a set of numerical examples.

Since transmission and generation are not an exact substitute for each other in a strict sense, any investor faces the dilemma of choosing which investment to consider or if a combination of both is better. This situation opens the door for competition between generation and transmission investments.

## B.1 Generation and Transmission Expansion Projects are not Interchangeable

In this section, it is shown that generation expansion is not an exact substitute for transmission and *vice versa*. Consider the two-node system shown in Figure B.1. Node 1 has cheap generation while Node 2 has two generators and three elastic demands. At Node 2, Generator 3 is less expensive than Generator 2.

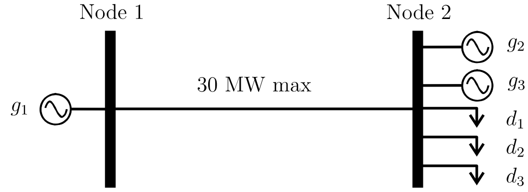


Figure B.1: Two-node system.

The demand and supply functions are as in Equations (4.2) and (4.4), respectively. All the pertinent data is given in Appendix C.

This numerical example considers losses as the sum of the squared power flow of the line times the resistance of the line over all the lines [3]. The objective function is the minimization of the social cost. The constraints are the usual system constraints. The solution is as shown in Figure B.2.

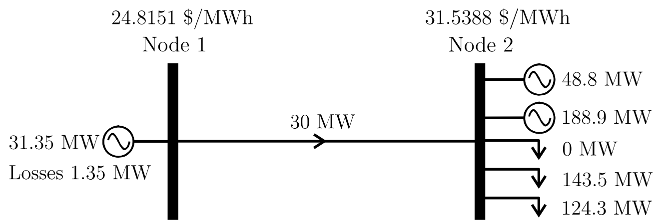


Figure B.2: Social Welfare 3311.959 $/h.

In the radial network an investor, either a new entrant or a participant of the market, faces the dilemma of either upgrading the transmission line or building a generator at Node 2 that can provide cheap energy. Suppose first that the capacity of the transmission line is doubled. When doubling the capacity of the transmission line, the susceptance is doubled while the resistance is halved. This is shown in Figure B.3.

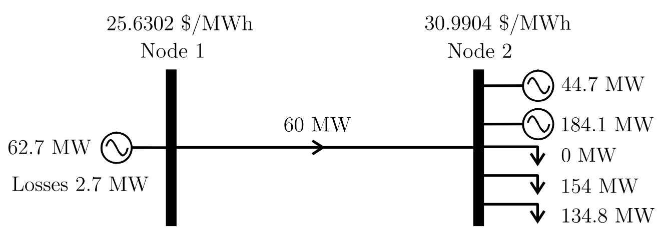


Figure B.3: Social Welfare 3459.168 $/h.

The social value of the investment can be tracked down by the changes in the consumer and producer surpluses, the congestion rent, and the cost of losses. These results are shown in Table B.1.

Now, instead of doubling the capacity of the line, assume that the investor decides to build a 30 MW generator at Node 2 of the same technology of the one located at Node 1. In this simple radial network, it is logical to substitute the expansion of the transmission

Table B.1: Social value of the investment, trans. exp., two-node system.

| | Surplus | | Δ |
|---|---|---|---|
| | Before | After | |
| Generator 1 | 12.753 | 51.012 | +38.259 |
| Generator 2 | 159.537 | 133.899 | −25.638 |
| Generator 3 | 2034.736 | 1932.44 | −102.296 |
| Demand 1 | 0.0 | 0.0 | 0.0 |
| Demand 2 | 535.283 | 616.862 | +81.579 |
| Demand 3 | 401.414 | 472.447 | +71.033 |
| | Congestion Rent | | Δ |
| | Before | After | |
| | 134.712 | 183.209 | +48.497 |
| | Cost of Losses | | Δ |
| | Before | After | |
| | 33.5 | 69.201 | +35.701 |
| Social Value of the Investment: 147.135 | | | |

All quantities in $/h

line with the building of a generator at the import constrained area since, in reality, such an expansion opens the possibility to import 30 more megawatts from the generator at Node 1. This situation is shown in Figure B.4.

It can be seen from Figures B.3 and B.4, that the nodal price, generation, and demand levels at Node 2 are exactly the same. However, the social welfare for the case of generation expansion is bigger than the one for transmission expansion. Even though strengthening the transmission line or building a new generator provide exactly the same 30 more megawatts of cheap energy, in the case of transmission expansion the congestion rent and the transmission losses are bigger due to the increased flow of power in the line making the generation expansion option more attractive from the point of view of the social welfare maximization. In the case of generation expansion, the cost of losses remains the same with respect to the congested case without any expansion. The details are shown in

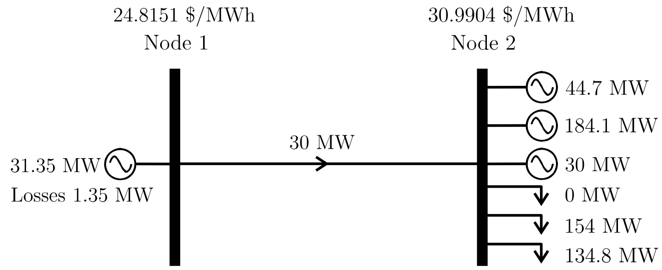


Figure B.4: Social Welfare 3518.198 $/h.

Table B.2.

Table B.2: Social value of the investment, gen. exp., two-node system.

| | Surplus | | Δ |
|---|---|---|---|
| | Before | After | |
| Generator 1 | 12.753 | 12.753 | 0.0 |
| Generator 4 | 0.0 | 198.012 | +198.012 |
| | Congestion Rent | | Δ |
| | Before | After | |
| | 134.712 | 118.258 | −16.454 |
| | Cost of Losses | | Δ |
| | Before | After | |
| | 33.5 | 33.5 | 0.0 |
| Social Value of the Investment: 206.236 | | | |

All quantities in $/h

It can be seen that, for this case, a generation expansion increases the social welfare more than the transmission expansion does and it actually reduces the congestion rent.

In the radial case it is easy to track down from which generating unit comes the increase of cheap energy due to the strengthening of the transmission line, in this case from generator $g_1$. Also, the change in the parameters of the transmission line does not alter the pattern of the power flow; before and after the transmission expansion, 100% of the flow goes from Node 1 to Node 2. However, these two conditions vanish in the presence of loop flows. Consider the three-node network shown in Figure B.5. All three transmission lines are identical. The data is shown in Appendix C.

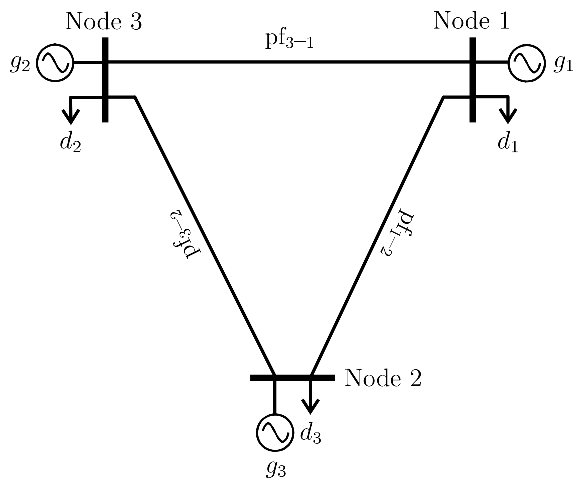


Figure B.5: Three-node system.

With the minimization of the social cost as the objective function, considering losses, taking Node 3 as the swing bus, and taking a limit of 30 MW at every line, one obtains the solution shown in Figure B.6.

It can be seen, from Figure B.6, that Line 1–2 is congested. If the capacity of transmission Line 1–2 is doubled, and considering the changes in the network parameters, one gets the results shown in Figure B.7.

As before, the social value of the investment can be tracked down by the changes in the consumer and producer surpluses, the congestion rent, and cost of losses. These results are shown in Table B.3.

From the transfer admittance matrix [3], one can see how the power flows in the transmission lines are related to the net power injections at every node excluding the swing bus. Equations (B.1) and (B.2) show the transfer admittance matrices before and after

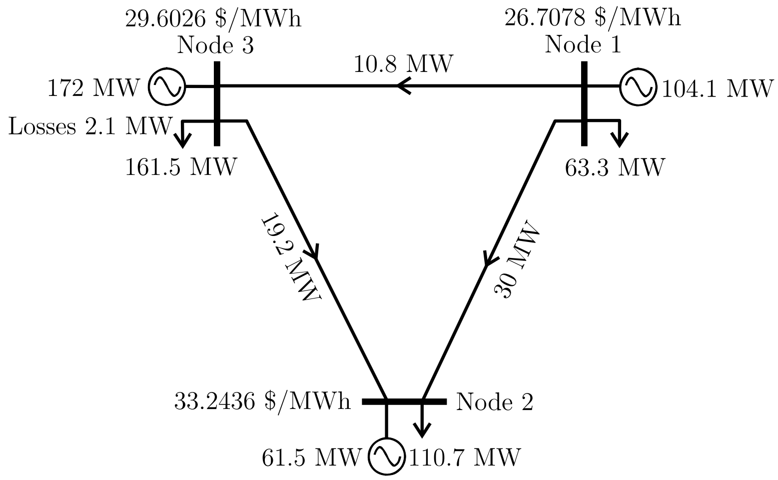


Figure B.6: Social Welfare: 3416.542 $/h.

the transmission expansion, respectively.

$$\boldsymbol{H}_B = \begin{bmatrix} \frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{3} & -\frac{2}{3} \\ -\frac{2}{3} & -\frac{1}{3} \end{bmatrix} \tag{B.1}$$

$$\boldsymbol{H}_A = \begin{bmatrix} \frac{2}{5} & -\frac{2}{5} \\ -\frac{2}{5} & -\frac{3}{5} \\ -\frac{3}{5} & -\frac{2}{5} \end{bmatrix} \tag{B.2}$$

Note how drastically the patterns of the power flows in the transmission lines change. For instance, before the expansion, the power that flows in the congested line, Line 1–2, is given by one third of the net power injected at Node 1 minus one third the net power injected at Node 2 whereas after the expansion, the power that flows in the line is given by two fifths of the net power injected at Node 1 minus two fifths of the net power injected at Node 2. Clearly, the changes in the parameters of the transmission lines must be considered in system planning studies.

Now assume that instead of strengthening the link between Nodes 1 and 2, an investor decides to build a generator at Node 2 of the same technology of the one at Node 1. The solution obtained is shown in Figure B.8.

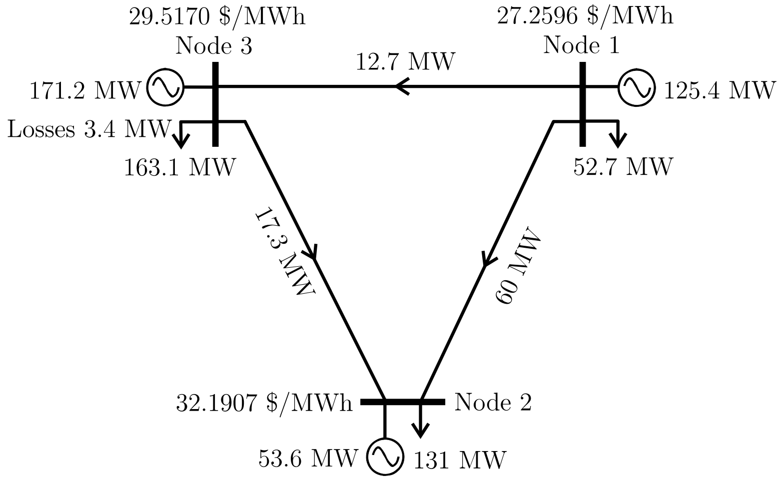


Figure B.7: Social Welfare: 3548.612 $/h.

In the same fashion, the social value of the investment can be tracked down by the changes in the consumer and producer surpluses, the congestion rent, and the cost of losses. These results are shown in Table B.4.

Comparing Figures B.7 and B.8, one can see that the nodal prices and the levels of generation and demand are similar but not the same. This is mainly because in the presence of loop flows, it is not easy to track down from which generator, or set of generators, comes the extra energy that flows to the import constrained area due to the transmission expansion. Under these circumstances, the building of a transmission line cannot be exactly replaced by the building of a generator at the import constrained area.

In essence, generation and transmission expansion projects are not interchangeable in a strict sense. The presence of loop flows in a network makes it difficult to track down from which generator, or set of generators, comes the extra energy that flows to the import constrained area due to a transmission expansion plan. The numerical examples show that, when planning a transmission expansion, it is not enough to modify the transmission capacity limits; the changes in the transmission line parameters, like the impedance and the resistance, have to be taken into account since these affect the transfer admittance matrix of the system which in turn drastically affects the power flows patterns of the system. Whenever considering a system expansion, generation and transmission investments must be treated as competitors and not as interchangeable options.

Table B.3: Social value of the investment, trans. exp., three-node system.

| | Surplus Before | Surplus After | Δ |
|---|---|---|---|
| Generator 1 | 141.003 | 204.310 | +63.307 |
| Generator 2 | 253.571 | 192.828 | −60.743 |
| Generator 3 | 1685.113 | 1670.023 | −15.09 |
| Demand 1 | 104.217 | 72.216 | −32.0 |
| Demand 2 | 318.617 | 446.09 | +127.473 |
| Demand 3 | 678.042 | 691.929 | +13.887 |
| | Congestion Rent Before | Congestion Rent After | Δ |
| | 174.219 | 170.455 | −3.764 |
| | Cost of Losses Before | Cost of Losses After | Δ |
| | 61.512 | 100.105 | +38.512 |
| Social Value of the Investment: 131.582 | | | |

All quantities in $/h

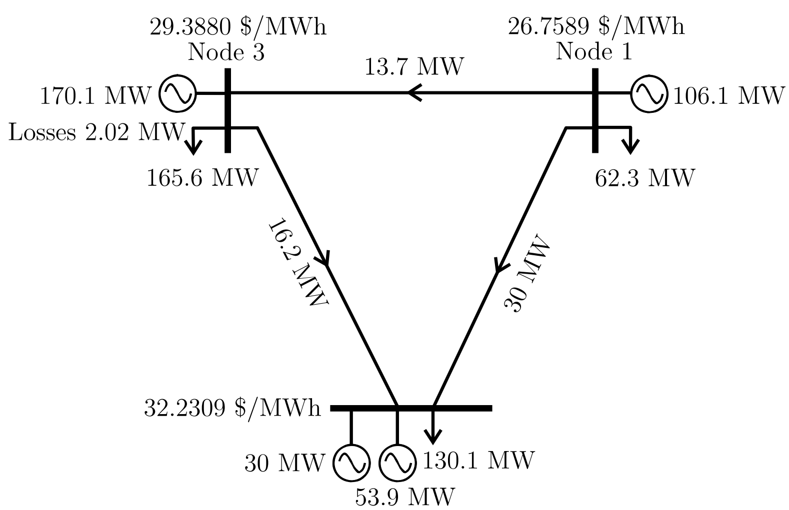


Figure B.8: Social Welfare: 3666.949 $/h.

Table B.4: Social value of the investment, gen. exp., three-node system.

| | Surplus | | $\Delta$ |
|---|---|---|---|
| | Before | After | |
| Generator 1 | 141.003 | 146.384 | +5.381 |
| Generator 2 | 253.571 | 195.097 | −58.474 |
| Generator 3 | 1685.113 | 1648.427 | −36.686 |
| Generator 4 | 0.0 | 235.227 | +235.227 |
| Demand 1 | 104.217 | 101.002 | −3.215 |
| Demand 2 | 318.617 | 440.582 | +121.965 |
| Demand 3 | 678.042 | 713.138 | +35.096 |
| | Congestion Rent | | $\Delta$ |
| | Before | After | |
| | 174.219 | 127.215 | −47.004 |
| | Cost of Losses | | $\Delta$ |
| | Before | After | |
| | 61.512 | 59.640 | −1.871 |
| Social Value of the Investment: 250.419 | | | |

All quantities in $/h

# Appendix C

# Test Systems Data

This appendix presents all the data for the numerical examples throughout this work. All the tables are self-contained in order to avoid excessive explanatory comments.

## C.1 Data for Section 3.4.3

Table C.1: Existing generating capacity data.

| Technology | $y_{et}{}^{\dagger}$ | $q_{et}{}^{\ddagger}$ | $E\{\alpha_{et}\}$ | $\sigma^2_{\alpha_{et}}$ |
|---|---|---|---|---|
| 1 | 500 | 0.1664 | 0.925 | 0.04087 |
| 3 | 200 | 0.5781 | 0.965 | 0.01327 |

† in MW, ‡ in \$M/MW-y

Table C.2: New generating capacity data.

| Technology | $x_{nt}$ † | $r_{nt}$ ‡ | $q_{nt}$ ‡ | $E\{\alpha_{nt}\}$ | $\sigma^2_{\alpha_{nt}}$ |
|---|---|---|---|---|---|
| 1 | 500 | 0.1737 | 0.1664 | 0.925 | 0.04087 |
| 2 | 100 | 0.0816 | 0.3723 | 0.965 | 0.01327 |
| 3 | 100 | 0.0405 | 0.5781 | 0.965 | 0.01327 |

† in MW, ‡ in \$M/MW-y

Table C.3: Operating modes and probabilities of foreseeable scenarios.

| Operating mode | $cf_m$ | Below Average† 30% | Average† 40% | Above Average† 30% |
|---|---|---|---|---|
| Base | 1.00 | 480 | 540 | 600 |
| Mid | 0.60 | 180 | 240 | 300 |
| Peak | 0.25 | 80 | 140 | 200 |

† in MW.

## C.2 Data for Section 3.5.2

Table C.4: Existing generating capacity data; six-node system.

| $et$ | $h$ | $y_{et,h}$ † | $q_{et}$ ‡ | $E\{\alpha_{et}\}$ | $\sigma^2_{\alpha_{et}}$ |
|---|---|---|---|---|---|
| 1 | 6 | 400 | 0.1664 | 0.925 | 0.04087 |
| 2 | 1 | 700 | 0.3723 | 0.965 | 0.01327 |
| 3 | 3 | 500 | 0.5781 | 0.965 | 0.01327 |

† in MW, ‡ in \$M/MW-y

Table C.5: New transmission lines data; six-node system.

| From: | To: | $R^{\dagger}$ | $X^{\dagger}$ | $\overline{f}_{i\text{-}j}{}^{\ddagger}$ | $c_{i\text{-}j}{}^{\star}$ | $E\{\alpha_{i\text{-}j}\}$ | $\sigma^2_{\alpha_{i\text{-}j}}$ |
|---|---|---|---|---|---|---|---|
| 2 | 6 | 0.075 | 0.30 | 100 | 3.0 | 1 | 0 |
| 4 | 6 | 0.080 | 0.30 | 100 | 3.0 | 1 | 0 |

† in p.u, 100 MVA base; ‡ in MW; ⋆ in \$M/y.

Table C.6: Existing transmission lines data; six-node system.

| From: | To: | $R^{\dagger}$ | $X^{\dagger}$ | $\overline{f}_{i\text{-}j}{}^{\ddagger}$ | $c_{i\text{-}j}{}^{\star}$ | $E\{\alpha_{i\text{-}j}\}$ | $\sigma^2_{\alpha_{i\text{-}j}}$ |
|---|---|---|---|---|---|---|---|
| 1 | 2 | 0.10 | 0.40 | 100 | 4.0 | 0.75 | 0.0085 |
| 1 | 4 | 0.15 | 0.60 | 80 | 5.6 | 0.45 | 0.0085 |
| 1 | 5 | 0.05 | 0.20 | 100 | 2.25 | 0.65 | 0.0085 |
| 2 | 3 | 0.05 | 0.20 | 100 | 2.25 | 0.75 | 0.0085 |
| 2 | 4 | 0.10 | 0.40 | 100 | 4.0 | 0.75 | 0.0085 |
| 3 | 5 | 0.05 | 0.20 | 100 | 2.25 | 0.85 | 0.0085 |

† in p.u, 100 MVA base; ‡ in MW; ⋆ in \$M/y.

Table C.7: Different foreseeable scenarios and their probabilities; six-node system.

| Node | Below Average† 30% | Average Demand† 40% | Above Average† 30% |
|---|---|---|---|
| 1 | 90 | 120 | 150 |
| 2 | 140 | 170 | 200 |
| 3 | 100 | 130 | 160 |
| 4 | 210 | 240 | 270 |
| 5 | 110 | 140 | 170 |
| 6 | 90 | 120 | 150 |

† in MW.

## C.3 Data for Section 3.6.2

The transmission lines data, existing and new, for the six-node system is as in Section C.2.

Table C.8: Existing generating capacity data, six-node system.

| $et$ | $h$ | $y_{et,h}$ † | $q_{et}$ ‡ | $E\{\alpha_{et}\}$ | $\sigma^2_{\alpha_{et}}$ |
|---|---|---|---|---|---|
| 1 | 1 | 500 | 0.1664 | 0.925 | 0.04087 |
| 2 | 3 | 300 | 0.3723 | 0.965 | 0.01327 |
| 3 | 6 | 400 | 0.5781 | 0.965 | 0.01327 |

† in MW, ‡ in \$M/MW-y

Table C.9: New generating capacity data, six-node system.

| $nt$ | $x_{nt}$ † | $r_{nt}$ ‡ | $q_{nt}$ ‡ | $\alpha_{nt}$ | $\sigma^2_{\alpha_{nt}}$ |
|---|---|---|---|---|---|
| 1 | 500 | 17.37 | 16.64 | 0.925 | 0.04087 |
| 2 | 300 | 8.16 | 37.23 | 0.965 | 0.01327 |
| 3 | 100 | 4.05 | 57.81 | 0.965 | 0.01327 |

† in MW, ‡ in \$M/MW-y

Table C.10: Below average level of demand, six-node system; 30%.

| Operating Mode† | Node | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 |
| B‡ | 80 | 80 | 80 | 80 | 80 | 80 |
| B‡ + M⋆ | 90 | 100 | 90 | 200 | 90 | 90 |
| B‡ + M⋆ + P∗ | 90 | 140 | 100 | 210 | 110 | 90 |

† in MW; ‡ Base, ⋆ Mid, and ∗ Peak.

Table C.11: Average level of demand, six-node system; 40%.

| Operating Mode† | Node | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 |
| B‡ | 90 | 90 | 90 | 90 | 90 | 90 |
| B‡ + M⋆ | 110 | 120 | 110 | 220 | 110 | 110 |
| B‡ + M⋆ + P∗ | 120 | 170 | 130 | 240 | 140 | 120 |

† in MW; ‡ Base, ⋆ Mid, and ∗ Peak.

Table C.12: Above average level of demand, six-node system; 30%.

| Operating Mode† | Node | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 |
| B‡ | 100 | 100 | 100 | 100 | 100 | 100 |
| B‡ + M⋆ | 130 | 140 | 130 | 240 | 130 | 130 |
| B‡ + M⋆ + P∗ | 150 | 200 | 160 | 270 | 170 | 150 |

† in MW; ‡ Base, ⋆ Mid, and ∗ Peak.

Table C.13: Existing transmission lines data, 21-node system.

| $i$ | $j$ | $R^{\dagger}$ | $X^{\dagger}$ | $\overline{f}_{i\text{-}j}{}^{\ddagger}$ | $c_{i\text{-}j}{}^{\star}$ | $E\{\alpha_{i\text{-}j}\}$ | $\sigma^2_{\alpha_{i\text{-}j}}$ |
|---|---|---|---|---|---|---|---|
| 1 | 2 | 0.0026 | 0.0139 | 200 | 1.47 | 0.95 | 0.0085 |
| 1 | 5 | 0.0218 | 0.0845 | 200 | 5.46 | 0.95 | 0.0085 |
| 1 | 3 | 0.0546 | 0.2112 | 200 | 12.39 | 0.95 | 0.0085 |
| 2 | 4 | 0.0328 | 0.1267 | 100 | 7.77 | 0.90 | 0.0085 |
| 2 | 6 | 0.0497 | 0.1920 | 100 | 11.34 | 0.90 | 0.0085 |
| 3 | 9 | 0.0308 | 0.1190 | 200 | 7.35 | 0.95 | 0.0085 |
| 3 | 13 | 0.0067 | 0.0519 | 200 | 12.48 | 0.90 | 0.0085 |
| 4 | 9 | 0.0268 | 0.1037 | 200 | 6.51 | 0.95 | 0.0085 |
| 5 | 10 | 0.0228 | 0.0883 | 200 | 5.67 | 0.95 | 0.0085 |
| 6 | 10 | 0.0139 | 0.0605 | 200 | 4.20 | 0.95 | 0.0085 |
| 7 | 8 | 0.0159 | 0.0614 | 200 | 4.20 | 0.95 | 0.0085 |
| 8 | 9 | 0.0427 | 0.1651 | 200 | 9.87 | 0.95 | 0.0085 |
| 8 | 10 | 0.0427 | 0.1651 | 200 | 9.87 | 0.95 | 0.0085 |
| 9 | 11 | 0.0061 | 0.0476 | 100 | 11.58 | 0.90 | 0.0085 |
| 9 | 12 | 0.0054 | 0.0418 | 100 | 10.38 | 0.90 | 0.0085 |
| 10 | 11 | 0.0061 | 0.0476 | 100 | 11.58 | 0.90 | 0.0085 |
| 10 | 21 | 0.0124 | 0.0966 | 100 | 21.78 | 0.90 | 0.0085 |
| 11 | 21 | 0.0111 | 0.0865 | 600 | 19.68 | 0.95 | 0.0085 |
| 12 | 14 | 0.0050 | 0.0389 | 600 | 9.79 | 0.95 | 0.0085 |
| 13 | 14 | 0.0022 | 0.0173 | 600 | 5.28 | 0.95 | 0.0085 |
| 13 | 19 | 0.0063 | 0.0490 | 600 | 11.88 | 0.95 | 0.0085 |
| 14 | 15 | 0.0033 | 0.0259 | 600 | 7.08 | 0.95 | 0.0085 |
| 14 | 17 | 0.0030 | 0.0231 | 600 | 6.48 | 0.95 | 0.0085 |
| 15 | 16 | 0.0018 | 0.0144 | 600 | 4.68 | 0.95 | 0.0085 |
| 15 | 20 | 0.0135 | 0.1053 | 600 | 23.58 | 0.95 | 0.0085 |
| 16 | 19 | 0.0033 | 0.0259 | 600 | 7.08 | 0.95 | 0.0085 |
| 17 | 18 | 0.0051 | 0.0396 | 600 | 9.93 | 0.95 | 0.0085 |
| 18 | 21 | 0.0028 | 0.0216 | 600 | 6.18 | 0.95 | 0.0085 |
| 19 | 20 | 0.0087 | 0.0678 | 600 | 15.78 | 0.95 | 0.0085 |

† in p.u, 100 MVA base; ‡ in MW; ⋆ in \$M/y.

Table C.14: Non-existing transmission lines data, 21-node system.

| $i$ | $j$ | $R^{\dagger}$ | $X^{\dagger}$ | $\overline{f}_{i\text{-}j}{}^{\ddagger}$ | $c_{i\text{-}j}{}^{\star}$ | $E\{\alpha_{i\text{-}j}\}$ | $\sigma^2_{\alpha_{i\text{-}j}}$ |
|---|---|---|---|---|---|---|---|
| 2 | 4 | 0.0328 | 0.1267 | 100 | 7.77 | 0.90 | 0.0085 |
| 2 | 6 | 0.0497 | 0.1920 | 100 | 11.34 | 0.90 | 0.0085 |
| 3 | 13 | 0.0067 | 0.0519 | 200 | 12.48 | 0.90 | 0.0085 |
| 9 | 11 | 0.0061 | 0.0476 | 100 | 11.58 | 0.90 | 0.0085 |
| 9 | 12 | 0.0054 | 0.0418 | 100 | 10.38 | 0.90 | 0.0085 |
| 10 | 11 | 0.0061 | 0.0476 | 100 | 11.58 | 0.90 | 0.0085 |
| 10 | 21 | 0.0124 | 0.0966 | 100 | 21.78 | 0.90 | 0.0085 |

† in p.u, 100 MVA base; ‡ in MW; ⋆ in \$M/y.

Table C.15: Existing generating capacity data, 21-node system.

| $h$ | $et$ | $y_{et,h}{}^{\dagger}$ | $q_{et}{}^{\ddagger}$ | $E\{\alpha_{et}\}$ | $\sigma^2_{\alpha_{et}}$ |
|---|---|---|---|---|---|
| 1 | 3 | 40 | 0.5781 | 0.965 | 0.01327 |
| 1 | 4 | 150 | 0.2558 | 0.925 | 0.04087 |
| 2 | 3 | 40 | 0.5781 | 0.965 | 0.01327 |
| 2 | 4 | 150 | 0.2558 | 0.925 | 0.04087 |
| 7 | 2 | 300 | 0.3723 | 0.965 | 0.01327 |
| 11 | 2 | 600 | 0.3723 | 0.965 | 0.01327 |
| 13 | 2 | 60 | 0.3723 | 0.965 | 0.01327 |
| 13 | 4 | 150 | 0.2558 | 0.925 | 0.04087 |
| 14 | 4 | 150 | 0.2558 | 0.925 | 0.04087 |
| 16 | 1 | 400 | 0.1664 | 0.925 | 0.04087 |
| 19 | 1 | 400 | 0.1664 | 0.925 | 0.04087 |
| 20 | 1 | 300 | 16.64 | 0.925 | 0.04087 |
| 21 | 4 | 660 | 0.2558 | 0.925 | 0.04087 |

† in MW, ‡ in \$M/MW-y

Table C.16: New generating capacity data, 21-node system.

| $nt$ | $x_{nt}{}^{\dagger}$ | $r_{nt}{}^{\ddagger}$ | $q_{nt}{}^{\ddagger}$ | $E\{\alpha_{nt}\}$ | $\sigma^2_{\alpha_{nt}}$ |
|---|---|---|---|---|---|
| 1 | 400 | 0.1737 | 0.1664 | 0.925 | 0.04087 |
| 2 | 100 | 0.0816 | 0.3723 | 0.965 | 0.01327 |
| 3 | 400 | 0.0405 | 0.5781 | 0.965 | 0.01327 |
| 4 | 100 | 0.1761 | 0.2558 | 0.925 | 0.04087 |

† in MW, ‡ in \$M/MW-y

Table C.17: Below average level of demand, 21-node system; 30%.

| Operating Mode† | Node | | | | |
|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 |
| B‡ | 47.50 | 42.50 | 78.75 | 32.50 | 31.25 |
| B‡ + M⋆ | 76.00 | 68.00 | 126.00 | 52.00 | 50.00 |
| B‡ + M⋆ + P∗ | 108.30 | 96.90 | 179.55 | 74.10 | 71.25 |
| | 6 | 7 | 8 | 9 | 10 |
| B‡ | 60.00 | 55.00 | 75.00 | 76.25 | 85.00 |
| B‡ + M⋆ | 96.00 | 88.00 | 120.00 | 122.00 | 136.00 |
| B‡ + M⋆ + P∗ | 136.80 | 125.40 | 171.00 | 173.85 | 193.80 |
| | 11 | 12 | 13 | 14 | 15 |
| B‡ | 116.25 | 85.00 | 138.75 | 43.75 | 0.00 |
| B‡ + M⋆ | 186.00 | 136.00 | 222.00 | 70.00 | 0.00 |
| B‡ + M⋆ + P∗ | 265.05 | 193.80 | 316.35 | 99.75 | 0.00 |
| | 16 | 17 | 18 | 19 | 20 |
| B‡ | 146.25 | 80.00 | 56.25 | 0.00 | 0.00 |
| B‡ + M⋆ | 234.00 | 128.00 | 90.00 | 0.00 | 0.00 |
| B‡ + M⋆ + P∗ | 333.45 | 182.40 | 128.25 | 0.00 | 0.00 |
| | 21 | | | | |
| B‡ | 0.00 | | | | |
| B‡ + M⋆ | 0.00 | | | | |
| B‡ + M⋆ + P∗ | 0.00 | | | | |

† in MW; ‡ Base, ⋆ Mid, and ∗ Peak.

Table C.18: Average level of demand, 21-node system; 40%.

| Operating Mode† | Node | | | | |
|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 |
| B‡ | 57.00 | 51.00 | 94.50 | 39.00 | 37.50 |
| B‡ + M⋆ | 91.20 | 81.60 | 151.20 | 62.40 | 60.00 |
| B‡ + M⋆ + P∗ | 129.96 | 116.28 | 215.46 | 88.92 | 85.50 |
| | 6 | 7 | 8 | 9 | 10 |
| B‡ | 72.00 | 66.00 | 90.00 | 91.50 | 102.00 |
| B‡ + M⋆ | 115.20 | 105.60 | 144.00 | 146.40 | 163.20 |
| B‡ + M⋆ + P∗ | 164.16 | 150.48 | 205.20 | 208.62 | 232.56 |
| | 11 | 12 | 13 | 14 | 15 |
| B‡ | 139.50 | 102.00 | 166.50 | 52.50 | 0.00 |
| B‡ + M⋆ | 223.20 | 163.20 | 266.40 | 84.00 | 0.00 |
| B‡ + M⋆ + P∗ | 318.06 | 232.56 | 379.62 | 119.70 | 0.00 |
| | 16 | 17 | 18 | 19 | 20 |
| B‡ | 175.50 | 96.00 | 67.50 | 0.00 | 0.00 |
| B‡ + M⋆ | 280.80 | 153.60 | 108.00 | 0.00 | 0.00 |
| B‡ + M⋆ + P∗ | 400.14 | 218.88 | 153.90 | 0.00 | 0.00 |
| | 21 | | | | |
| B‡ | 0.00 | | | | |
| B‡ + M⋆ | 0.00 | | | | |
| B‡ + M⋆ + P∗ | 0.00 | | | | |

† in MW; ‡ Base, ⋆ Mid, and ∗ Peak.

Table C.19: Above average level of demand, 21-node system; 30%.

| Operating Mode† | Node | | | | |
|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 |
| B‡ | 65.55 | 58.65 | 108.67 | 44.85 | 43.12 |
| B‡ + M⋆ | 104.88 | 93.84 | 173.88 | 71.76 | 69.00 |
| B‡ + M⋆ + P∗ | 149.45 | 133.72 | 247.77 | 102.25 | 98.32 |
| | 6 | 7 | 8 | 9 | 10 |
| B‡ | 82.80 | 75.90 | 103.50 | 105.22 | 117.30 |
| B‡ + M⋆ | 132.48 | 121.44 | 165.60 | 168.36 | 187.68 |
| B‡ + M⋆ + P∗ | 188.78 | 173.05 | 235.98 | 239.91 | 267.44 |
| | 11 | 12 | 13 | 14 | 15 |
| B‡ | 160.42 | 117.30 | 191.47 | 60.37 | 0.00 |
| B‡ + M⋆ | 256.68 | 187.68 | 306.36 | 96.60 | 0.00 |
| B‡ + M⋆ + P∗ | 365.76 | 267.44 | 436.56 | 137.65 | 0.00 |
| | 16 | 17 | 18 | 19 | 20 |
| B‡ | 201.82 | 110.40 | 77.62 | 0.00 | 0.00 |
| B‡ + M⋆ | 322.92 | 176.64 | 124.20 | 0.00 | 0.00 |
| B‡ + M⋆ + P∗ | 460.16 | 251.71 | 176.98 | 0.00 | 0.00 |
| | 21 | | | | |
| B‡ | 0.00 | | | | |
| B‡ + M⋆ | 0.00 | | | | |
| B‡ + M⋆ + P∗ | 0.00 | | | | |

† in MW; ‡ Base, ⋆ Mid, and ∗ Peak.

## C.4 Data for Section 4.5

The transmission network data for the six- and 21-node systems is as in Sections C.2 and C.3, respectively. The only difference is the transmission capacity limits; all transmission lines have a capacity limit of 50 MW.

Table C.20: Cost function data; three-node system.

| Node $\nu$ | $E\{a_{1,\nu}{}^{\dagger}\}$ | $E\{a_{2,\nu}{}^{\ddagger}\}$ | $\sigma^2_{a_{1,\nu}}$ | $\sigma^2_{a_{2,\nu}}$ | $\sigma_{a_{1,\nu},a_{2,\nu}}$ |
|---|---|---|---|---|---|
| 1 | 24 | 0.026 | 2.4 | 0.0026 | 0.0624 |
| 2 | 25 | 0.134 | 2.5 | 0.0134 | 0.3350 |
| 3 | 10 | 0.114 | 1.0 | 0.0114 | 0.1140 |

† in \$/MWh, ‡ in \$/MW$^2$h.

Table C.21: Benefit function data; three-node system.

| Node $\nu$ | $E\{b_{1,\nu}{}^{\dagger}\}$ | $E\{b_{2,\nu}{}^{\ddagger}\}$ | $\sigma^2_{b_{1,\nu}}$ | $\sigma^2_{b_{2,\nu}}$ | $\sigma_{b_{1,\nu},b_{2.\nu}}$ |
|---|---|---|---|---|---|
| 1 | 30 | 0.052 | 3.0 | 0.0052 | −0.1560 |
| 2 | 39 | 0.052 | 3.9 | 0.0052 | −0.2028 |
| 3 | 38 | 0.052 | 3.8 | 0.0052 | −0.1976 |

† in \$/MWh, ‡ in \$/MW$^2$h.

Table C.22: Transmission line data.

| From: | To: | $R^\dagger$ | $X^\dagger$ | $\overline{f}_{i\text{-}j}{}^\ddagger$ |
|---|---|---|---|---|
| 1 | 2 | 0.0015 | 0.3 | 30 |
| 3 | 1 | 0.0015 | 0.3 | 30 |
| 3 | 2 | 0.0015 | 0.3 | 30 |

† in p.u, 100 MVA base; ‡ in MW.

Table C.23: Cost function data; six-node system.

| Node $\nu$ | $E\{a_{1,\nu}{}^\dagger\}$ | $E\{a_{2,\nu}{}^\ddagger\}$ | $\sigma^2_{a_{1,\nu}}$ | $\sigma^2_{a_{2,\nu}}$ | $\sigma_{a_{1,\nu},a_{2,\nu}}$ |
|---|---|---|---|---|---|
| 1 | 24 | 0.026 | 2.4 | 0.0026 | 0.0624 |
| 2 | 25 | 0.134 | 2.5 | 0.0134 | 0.3350 |
| 3 | 10 | 0.114 | 1.0 | 0.0114 | 0.1140 |

† in \$/MWh, ‡ in \$/MW$^2$h.

Table C.24: Benefit function data; six-node system.

| Node $\nu$ | $E\{b_{1,\nu}{}^\dagger\}$ | $E\{b_{2,\nu}{}^\ddagger\}$ | $\sigma^2_{b_{1,\nu}}$ | $\sigma^2_{b_{2,\nu}}$ | $\sigma_{b_{1,\nu},b_{2.\nu}}$ |
|---|---|---|---|---|---|
| 1 | 30 | 0.052 | 3.0 | 0.0052 | −0.1560 |
| 2 | 39 | 0.052 | 3.9 | 0.0052 | −0.2028 |
| 3 | 37 | 0.052 | 3.7 | 0.0052 | −0.1924 |
| 4 | 36 | 0.052 | 3.6 | 0.0052 | −0.1872 |
| 5 | 35 | 0.052 | 3.5 | 0.0052 | −0.1820 |
| 6 | 38 | 0.052 | 3.8 | 0.0052 | −0.1976 |

† in \$/MWh, ‡ in \$/MW$^2$h.

Table C.25: Cost function data; 21-node system.

| Node $\nu$ | $E\{a_{1,\nu}{}^{\dagger}\}$ | $E\{a_{2,\nu}{}^{\ddagger}\}$ | $\sigma^2_{a_{1,\nu}}$ | $\sigma^2_{a_{2,\nu}}$ | $\sigma_{a_{1,\nu},a_{2,\nu}}$ |
|---|---|---|---|---|---|
| 1 | 24 | 0.026 | 2.4 | 0.0026 | 0.0624 |
| 2 | 25 | 0.134 | 2.5 | 0.0134 | 0.3350 |
| 7 | 10 | 0.114 | 1.0 | 0.0114 | 0.1140 |
| 11 | 24 | 0.026 | 2.4 | 0.0026 | 0.0624 |
| 13 | 25 | 0.134 | 2.5 | 0.0134 | 0.3350 |
| 14 | 10 | 0.114 | 1.0 | 0.0114 | 0.1140 |
| 16 | 24 | 0.026 | 2.4 | 0.0026 | 0.0624 |
| 19 | 25 | 0.134 | 2.5 | 0.0134 | 0.3350 |
| 20 | 10 | 0.114 | 1.0 | 0.0114 | 0.1140 |
| 21 | 10 | 0.114 | 1.0 | 0.0114 | 0.1140 |

† in \$/MWh, ‡ in \$/MW$^2$h.

Table C.26: Benefit function data; 21-node system.

| Node $\nu$ | $E\{b_{1,\nu}{}^{\dagger}\}$ | $E\{b_{2,\nu}{}^{\ddagger}\}$ | $\sigma^2_{b_{1,\nu}}$ | $\sigma^2_{b_{2,\nu}}$ | $\sigma_{b_{1,\nu},b_{2.\nu}}$ |
|---|---|---|---|---|---|
| 1 | 30 | 0.052 | 3.0 | 0.0052 | −0.1560 |
| 2 | 39 | 0.052 | 3.9 | 0.0052 | −0.2028 |
| 3 | 37 | 0.052 | 3.7 | 0.0052 | −0.1924 |
| 4 | 36 | 0.052 | 3.6 | 0.0052 | −0.1872 |
| 5 | 35 | 0.052 | 3.5 | 0.0052 | −0.1820 |
| 6 | 38 | 0.052 | 3.8 | 0.0052 | −0.1976 |
| 7 | 30 | 0.052 | 3.0 | 0.0052 | −0.1560 |
| 8 | 39 | 0.052 | 3.9 | 0.0052 | −0.2028 |
| 9 | 37 | 0.052 | 3.7 | 0.0052 | −0.1924 |
| 10 | 36 | 0.052 | 3.6 | 0.0052 | −0.1872 |
| 11 | 35 | 0.052 | 3.5 | 0.0052 | −0.1820 |
| 12 | 38 | 0.052 | 3.8 | 0.0052 | −0.1976 |
| 13 | 30 | 0.052 | 3.0 | 0.0052 | −0.1560 |
| 14 | 39 | 0.052 | 3.9 | 0.0052 | −0.2028 |
| 15 | 37 | 0.052 | 3.7 | 0.0052 | −0.1924 |
| 16 | 36 | 0.052 | 3.6 | 0.0052 | −0.1872 |
| 17 | 35 | 0.052 | 3.5 | 0.0052 | −0.1820 |
| 18 | 38 | 0.052 | 3.8 | 0.0052 | −0.1976 |

† in \$/MWh, ‡ in \$/MW$^2$h.

## C.5 Data for Section B.1

The transmission network data for the three-node system is as in Section C.4.

Table C.27: Cost and Benefit functions data; all systems.

| Generator | $a_{1,\nu}$ | $a_{2,\nu}$ | Demand | $b_{1,\nu}$ | $b_{2,\nu}$ |
|---|---|---|---|---|---|
| 1 | 24 | 0.026 | 1 | 30 | 0.052 |
| 2 | 25 | 0.134 | 2 | 39 | 0.052 |
| 3 | 10 | 0.114 | 3 | 38 | 0.052 |

† in \$/MWh, ‡ in \$/MW$^2$h.

The transmission line data is shown in Table C.28.

Table C.28: Transmission line data; two-node system.

| From: | To: | $R^\dagger$ | $X^\dagger$ | $\overline{f}_{i\text{-}j}{}^\ddagger$ |
|---|---|---|---|---|
| 1 | 2 | 0.0015 | 0.3 | 30 |

† in p.u, 100 MVA base; ‡ in MW.

# Appendix D

# Large Test Systems

This appendix presents two large-scale systems in order to further validate the models presented in this thesis; these systems are based on the IEEE 57- and 118-node systems [87]. All the models are implemented using the optimization software GAMS using the SBB and the MINOS solvers [42]. These models are solved using the NEOS server for optimization [88]; in all cases, including the problems solved in Chapters 3 and 4, an integer solution is obtained for the power system expansion problems and a locally optimal solution is obtained for the power pool electricity market problems. For the power system expansion problems, the most time consuming part is the branch and bounding process; solving the nonlinear part poses no challenge.

## D.1 A 57-Node System

The 57-node system has a mixture of four different generating technologies and it has eighty transmission lines. For the generation and transmission expansion case, it is possible to add generating capacity at every node; one can choose from among four different technologies. Even though there are no new rights-of-way, it is possible to add up to five new circuits between every pair of nodes that already have an interconnection. Three operating modes are considered with three foreseeable scenarios each. These scenarios happen with a known probability. All the random variables are assumed to be standard normally distributed.

Table D.1 shows some results when the two-stage stochastic model described by Equations (3.71), (3.76)–(3.79), (3.60), (3.61), (2.46), (2.54), and (3.64)–(3.68) is solved. The parameters $\theta_r$, $\beta_t$, and $\beta_g$ are allowed to vary. Node 1 is taken as the slack node and the budget constraint is not binding.

The general tendency observed is that as the risk aversion increases, the variance (standard deviation) of the generation cost decreases and the overall cost increases. In like manner, as the probability of satisfying the chance constraints increases, the general tendency is that the overall cost also increases.

Table D.2 shows some results when the model shown in Equations (4.52)–(4.54) is solved taking Node 1 as the slack node. The intercepts and slopes of all the supply and demand functions are taken as standard normally distributed random variables. There are four suppliers and forty-two consumers.

Just as intended, by increasing the value of $\theta_r$ the social cost variance decreases. For this particular case, the nodal price increases and the generation decreases at the slack node as $\theta_r$ increases.

Table D.1: Annualized generating cost, gen. & trans. exp., 57-node system.

| Parameter | Mean† | Std. Deviation† | Overall Cost‡ |
|---|---|---|---|
| $\theta_r = 0.00$ | 210.6300 | 67.4844 | 312.7500 |
| $\theta_r = 0.02$ | 158.5390 | 36.0964 | 321.9790 |
| $\theta_r = 0.03$ | 129.4960 | 24.6157 | 337.9360 |
| $\theta_r = 0.04$ | 179.3230 | 19.0941 | 361.6030 |
| $\theta_r = 0.10$ | 200.9350 | 7.6376 | 362.7250 |
| $\theta_r = 0.03$, $\beta_t = 0$ | | | |
| $\approx 76\%; \beta_g = 0.7$ | 150.6730 | 25.4588 | 365.6230 |
| $\approx 84\%; \beta_g = 1.0$ | 197.8930 | 25.4588 | 378.9430 |
| $\approx 90\%; \beta_g = 1.3$ | 210.9800 | 25.4588 | 390.7400 |
| $\approx 96\%; \beta_g = 1.74$ | 177.5320 | 25.4588 | 418.6120 |
| $\theta_r = 0.03$, $\beta_g = 0$ | | | |
| $\approx 76\%; \beta_t = 0.7$ | 170.2940 | 25.4588 | 343.1390 |
| $\approx 84\%; \beta_t = 1.0$ | 179.6750 | 25.4588 | 334.9550 |
| $\approx 90\%; \beta_t = 1.3$ | 196.0680 | 25.4588 | 352.6380 |
| $\approx 96\%; \beta_t = 1.74$ | 175.0800 | 25.4588 | 336.8700 |
| $\beta_g = 1.3$, $\beta_t = 1.3$ | | | |
| $\theta_r = 0.00$ | 237.7020 | 72.4940 | 347.9820 |
| $\theta_r = 0.01$ | 138.9420 | 41.3971 | 371.8620 |
| $\theta_r = 0.02$ | 191.5360 | 38.1881 | 371.296 |
| $\theta_r = 0.03$ | 186.0190 | 25.4588 | 418.9390 |
| $\theta_r = 0.04$ | 244.6230 | 19.0941 | 408.0630 |
| $\theta_r = 0.10$ | 214.8920 | 7.6376 | 447.8120 |

† in \$M-y; ‡ Investment cost plus annualized generating cost, in \$M-y.

Table D.2: Randomness in supply and demand intercepts–slopes, 57-node system.

| Parameter | $\theta_r = 0$ | $\theta_r = 0.0001$ | $\theta_r = 0.001$ | $\theta_r = 0.01$ |
|---|---|---|---|---|
| $E\{SC\}^{\dagger}$ | $-8050.29$ | $-7781.21$ | $-6169.20$ | $-3394.88$ |
| ${\sigma_{SC}^2}^{\ddagger}$ | 13078.00 | 6135.20 | 1140.2 | 95.24 |
| ${\rho_1}^{*}$ | 30.37 | 32.25 | 35.94 | 36.44 |
| ${g_1}^{\star}$ | 178.70 | 152.48 | 99.08 | 47.61 |
| ${d_1}^{\star}$ | 0.00 | 0.00 | 0.00 | 0.00 |

$^{\dagger}$ in \$/h, $^{\ddagger}$ $\times 10^3$ in $\$^2/\mathrm{h}^2$, $^{*}$ in \$/MWh, and $^{\star}$ in MW

## D.2 A 118-Node System

The 118-node system has fifty-three generating units with a mixture of four different technologies. The system has one hundred and eighty-six transmission lines. It is possible to add generating capacity at every node and there are four different technologies to choose from. It is also possible to build up to five new circuits between every pair of nodes that already have an interconnection; there are no new rights-of-way. Three operating modes are considered; each one of which has three different foreseeable scenarios. These foreseeable scenarios happen with a known probability. All the random variables are taken as standard normally distributed.

Now, solving the two-stage stochastic model described by Equations (3.71), (3.76)–(3.79), (3.60), (3.61), (2.46), (2.54), and (3.64)–(3.68) for the 118-node system one gets the results shown in Table D.3. Node 69 is taken as the slack node. The execution time ranges from 0.755 to 31.192 seconds when solved using the NEOS server for optimization. Some of the model statistics are: 22,304 constraints, 11,178 non-linear variables, and 658 discrete variables.

From the results one can see that as the risk aversion increases, larger values of $\theta_r$, the variance (standard deviation) of the generation cost decreases and the overall cost increases. The overall cost also increases as the probability of satisfying the probabilistic constraints increases.

Table D.4 shows some results when the model shown in Equations (4.52)–(4.54) is solved with Node 69 as the slack node. Just like before, the intercepts and slopes of all the supply and demand functions are taken as standard normally distributed random variables. There are fifty-three suppliers and ninety-two consumers.

It can be seen that, as $\theta_r$ increases, the social cost variance decreases. It also can be seen that the generation level at the slack node decreases as the risk aversion increases.

Table D.3: Annualized generating cost, gen. & trans. exp., 118-node system.

| Parameter | Mean† | Std. Deviation† | Overall Cost‡ |
|---|---|---|---|
| $\theta_r = 0.000$ | 595.398 | 75.9100 | 603.558 |
| $\theta_r = 0.002$ | 592.530 | 73.5726 | 608.850 |
| $\theta_r = 0.010$ | 553.937 | 51.3344 | 623.417 |
| $\theta_r = 0.020$ | 564.638 | 38.1881 | 634.118 |
| $\theta_r = 0.0,\ \beta_t = 0$ | | | |
| $\approx 76\%; \beta_g = 0.7$ | 584.317 | 76.7450 | 659.077 |
| $\approx 84\%; \beta_g = 1.0$ | 667.507 | 85.8750 | 691.987 |
| $\approx 90\%; \beta_g = 1.74$ | 710.363 | 101.3930 | 779.843 |
| $\theta_r = 0.01,\ \beta_t = 0$ | | | |
| $\approx 90\%; \beta_g = 1.3$ | 713.249 | 60.9740 | 756.629 |
| $\approx 96\%; \beta_g = 1.74$ | 673.669 | 61.1553 | 812.629 |
| $\theta_r = 0.0,\ \beta_g = 0$ | | | |
| $\approx 76\%; \beta_t = 0.7$ | 599.466 | 75.4894 | 607.626 |
| $\approx 84\%; \beta_t = 1.0$ | 601.523 | 76.0043 | 609.683 |
| $\approx 90\%; \beta_t = 1.3$ | 604.066 | 75.6499 | 612.226 |
| $\approx 96\%; \beta_t = 1.74$ | 604.775 | 77.2193 | 612.935 |
| $\theta_r = 0.01,\ \beta_g = 0$ | | | |
| $\approx 76\%; \beta_t = 0.7$ | 551.369 | 50.5973 | 620.849 |
| $\approx 90\%; \beta_t = 1.3$ | 553.870 | 50.9000 | 624.820 |
| $\theta_r = 0.02,\ \beta_g = 0$ | | | |
| $\approx 90\%; \beta_t = 1.3$ | 570.868 | 38.1881 | 640.348 |
| $\beta_g = 0.7,\ \beta_t = 0.7$ | | | |
| $\theta_r = 0.01$ | 603.539 | 55.4356 | 674.489 |
| $\beta_g = 1.3,\ \beta_t = 1.3$ | | | |
| $\theta_r = 0.01$ | 670.976 | 60.6844 | 741.926 |

† in $M-y; ‡ Investment cost plus annualized generating cost, in $M-y.

Table D.4: Randomness in supply and demand intercepts–slopes, 118-node system.

| Parameter | $\theta_r = 0$ | $\theta_r = 0.0001$ | $\theta_r = 0.001$ | $\theta_r = 0.01$ | $\theta_r = 0.1$ |
|---|---|---|---|---|---|
| $E\{SC\}^{\dagger}$ | −72380.00 | −69780.00 | −53970.00 | −27800.00 | −9710.45 |
| ${\sigma^2_{SC}}^{\ddagger}$ | 133870.00 | 60138.00 | 11122.00 | 824.84 | 36.17 |
| ${\rho_{69}}^{*}$ | 30.02 | 30.66 | 30.92 | 30.06 | 28.93 |
| ${g_{69}}^{\star}$ | 175.66 | 144.54 | 88.66 | 41.96 | 16.21 |

$^{\dagger}$ in \$/h, $^{\ddagger}$ $\times 10^3$ in \$$^2$/h$^2$, $^{*}$ in \$/MWh, and $^{\star}$ in MW

# Bibliography


[1] R. R. Nelson. Uncertainty, prediction, and competitive equilibrium. *The Quarterly Journal of Economics*, 75(1):41–62, February 1961.

[2] B. R. Wollemberg and A. J. Wood. *Power Generation Operation and Control*. John Wiley & Sons, 1996.

[3] F. C. Schweppe, M. C. Caraminis, R. D. Tabors, and R. E. Bohn. *Spot pricing of electricity*. Kluwer Academic Publishers, February 1988.

[4] H. G. Stoll. *Least-Cost Electric Utility Planning*. John Wiley & Sons, 1989.

[5] J. R. Birge and F. Louveaux. *Introduction to Stochastic Programming*. Springer-Verlag, 1997.

[6] K. Leong and R. Siddiqi. *Energy Risk Management*, chapter "Value at Risk for Power Markets", pages 157–178. McGraw-Hill, 1998.

[7] R. S. Pindyck. Irreversibility, uncertainty, and investment. *Journal of Economic Literature*, 29(3):1110–1148, September 1991.

[8] G. P. Prastacos. Optimal sequential investment decisions under conditions of uncertainty. *Management Science*, 29(1):118–134, January 1983.

[9] S. K. Gupta and J. Rosenhead. Robustness in sequential investment decisions. *Management Science*, 15(2):B18–B29, October 1968.

[10] P. L. Joskow. Patterns of transmission investment, March 2005. http://econ-www.mit.edu/faculty/pjoskow/papers,.

[11] J. Park, J. Kim, and K. Y. Lee. Generation expansion planning in a competitive environment using a genetic algorithm. In *Proceedings of the 2002 IEEE Power Engineering Society Summer Meeting*, volume 3, pages 1169–1172, 2002.

[12] A. Botterud, M. D Ilic, and I. Wangensteen. Optimal investments in power generation under centralized and decentralized decision making. *IEEE Trans. on Power Systems*, 20(1):254–263, February 2005.

[13] P. Joskow and J. Tirole. Merchant transmission investment. *Journal of industrial economics*, 53(2):233–264, June 2005.

[14] M. O. Buygi, G. Balzer, H. M. Shanechi, and M. Shahidehpour. Market-based transmission expansion planning. *IEEE Trans. on Power Systems*, 19(4):2060–2067, November 2004.

[15] M. O. Buygi, H. M. Shanechi, G. Balzer, M. Shahidehpour, and N. Pariz. Newtwork planning in unbundled power systems. *IEEE Trans. on Power Systems*, 21(3):1379–1387, August 2006.

[16] T. Kristiansen. Merchant transmission expansion based on financial transmission rights. *IEEE Proc.* $38^{th}$ *Annual International Conference on System Sciences HICSS '05*, pages 1–10, January 2005.

[17] A. K. David and F. Wen. Transmission planning and investment under competitive electricity market environment. In *IEEE Proccedings of the 2001 Power Engineering Society Summer Meeting*, volume 3, pages 1725–1730, July 2001.

[18] M. Barmack, P. Griffes, E. Kahn, and S. Oren. Performance incentives for transmission. *The Electricity Journal*, 16(3):9–22, April 2003.

[19] G. B. Shrestha and P. A. J. Fonseka. Congestion-driven transmission expansion in competitive markets. *IEEE Trans. on Power Systems*, 19(3):1658–1665, August 2004.

[20] G. B. Shrestha and P. A. J. Fonseka. Transmission planning in competitive power markets considering the uncertainties in market operation. In *Proceedings of the 2004*

*Australasian Universities Power Engineering Conference (CD)*, pages 1–6, The University of Queensland, Brisbane, Australia, 26–29 September 2004.

[21] S. Hunt. *Making Competition Work in Electricity.* John Wiley & Sons, 2002.

[22] S. Stoft. *Power System Economics Designing Markets for Electricity.* IEEE Press Wiley-Interscience, 2000.

[23] R. Fang and D. J. Hill. A new strategy for transmission expansion in competitive electricity markets. *IEEE Trans. on Power Systems*, 18(1):374–380, February 2003.

[24] H. A. Gil, E. L. da Silva, and F. D. Galiana. Modeling competition in transmission expansion. *IEEE Trans. on Power Systems*, 17(4):1043–1049, November 2002.

[25] J. McDonald. *Modern Power System Planning*, chapter "The forecasting of growth of demand for electrical energy", pages 1–88. McGraw-Hill, 1994.

[26] X. Wang. *Modern Power System Planning*, chapter "Generation planning", pages 308–374. McGraw-Hill, 1994.

[27] P. Joskow and R. Schmalensee. *Markets for Power An Analysis of Electric Utility Deregulation.* MIT Press, 1983.

[28] S. Pokharel and K. Ponnambalam. Investment planning for electricity generation expansion. *International Journal of Energy Research*, 21:185–194, February 1997.

[29] G. L. Torres. *Nonlinear Optimal Power Flow by Interior and Non-Interior Point Methods.* PhD thesis, University of Waterloo, Waterloo, Ontario, Canada, 1998.

[30] L. L. Garver. Transmission network estimation using linear programming. PAS-89(7):1688–1697, September/October 1970.

[31] R. Villasana, L. L. Garver, and S. J. Salon. Transmission network planning using linear programming. *IEEE Trans. on Power Apparatus and Systems*, PAS-104(2):349–356, February 1985.

[32] R. Romero, A. Monticelli, A. Garcia, and S. Haffner. Test system and mathematical models for transmission network expansion planning. *IEE Proc. Gen. Ttrans. Distrib.*, 149(1):27–36, January 2002.

[33] R. Romero, C. Rocha, J. R. S. Mantovani, and I. G. Sánchez. Constructive heuristic algorithm for the dc model in network transmission expansion planning. *IEE Proc. Gen. Ttrans. Distrib.*, 152(2):277–282, March 2005.

[34] P. Kundur. *Power System Stability and Control*. Electric Power Research Institute, 1994.

[35] J. Alvarez, K. Ponnambalam, and V. H. Quintana. Generation and transmission expansion under risk using stochastic programming. *IEEE Transactions on Power Systems*, 22(3):1369–1378, August 2007.

[36] H. M. Markowitz. *Portfolio Selection, Efficient Diversification of Investments*. Yale University Press, 1959.

[37] H. M. Markowitz. Portfolio selection. *The Journal of Finance*, 7(1):77–91, March 1952.

[38] D. G. Luenberger. *Investment Science*. Oxford University Press, 1998.

[39] K. Ponnambalam and J. L. Zhang. Stochastic control for risk under deregulated electricity market — a case study using a new formulation. *Canadian Journal of Civil Engineering*, 32(4):719–725, August 2005.

[40] S. Manandhar, A. Tarim, and T. Walsh. Scenario-based stochastic constraint programming. *International Joint Conferences on Artificial Intelligence*, page paper 214, August 2003.

[41] A. Papoulis. *Probability, Random Variables and Stochastic Process*. McGraw-Hill, 1991.

[42] http://www.gams.com.

[43] J. Alvarez, K. Ponnambalam, and V. H. Quintana. Transmission expansion under risk using stochastic programming. In *IEEE Proceedings of the* $9^{th}$ *International Conference on Probabilistic Methods Applied to Power Systems (CD)*, pages 1–7, Kungliga Tekniska Högskolan (KTH), Stockholm, Sweden, 11–15 June 2006.

[44] F. Evans, J. M. Zolezzi, and H. Rudnick. Cost assingment model for electrical transmission system expansion: An approach through the kernel theory. *IEEE Trans. on Power Systems*, 18(2):625–632, May 2003.

[45] H. Saadat. *Power System Analysis*. McGraw-Hill, 1999.

[46] Y. Ermoliev and R. J-B Wets. *Numerical Techniques for Stochastic Optimization*, chapter "Stochastic Programming, an Introduction", pages 1–32. Springer-Verlag, 1988.

[47] Reliability Test System Task Force. IEEE reliability test system. PAS-98(6):2047–2054, November/December 1979.

[48] J. Alvarez, K. Ponnambalam, and V. H. Quintana. The investor's dilemma: Generation or transmission expansion? In *IEEE Proceedings of the* $20^{th}$ *Canadian Conference on Electrical and Computer Engineering CCECE 2007*, pages 1281–1285, University of British Columbia, Vancouver, Canada, 22–26 April 2007.

[49] P. Bresesti, M. Gallanti, and D. Lucarella. Market-based genertaion and transmission expansions in the competitive market. In *Proceedings of the 2003 IEEE Power Engineering Society General Meeting*, volume 1, pages 459–462, July 2003.

[50] W. W. Hogan. Contract networks for electric power transmission. *Journal of Regulatory Economics*, 4(3):211–242, September 1992.

[51] G. Bautista, M. F. Anjos, and A. Vannelli. Beyond the use of linear approximations for modelling nash-cournot equilibria. In *IEEE Proceedings of the 2007 Power Tech Conference (CD)*, pages 1–6, École Polytechnique Fédérale de Lausanne, Lausanne, Switzerland, 1–5 July 2007.

[52] D. P. Baron. Price uncertainty, utility, and industry equilibrium in pure competition. *International Economic Review*, 11(3):463–480, October 1970.

[53] P. D. Klemperer and M. A. Meyer. Supply function equilibria in oligopoly under uncertainty. *Econometrica*, 57(6):1243–1277, November 1989.

[54] R. S. Pindyck and D. L. Rubinfeld. *Microeconomics*. Macmillan, 2nd edition, 1992.

[55] G. Bautista. *Alternative Models to Analyze Market Power and Financial Transmission Rights in Electricity Markets*. PhD thesis, University of Waterloo, Waterloo, Ontario, Canada, 2005.

[56] S. S. Oren, P. Spiller, P. Varaiya, and F. Wu. Folk theorems on transmission access: Proofs and counter examples. Working Paper, February 1995. http://www.ieor.berkeley.edu/~oren/pubs.htm.

[57] M. Caramanis. Investment decisions and lon-term planning under electricity spot pricing. *IEEE Trans. on Power Apparatus and Systems*, PAS-101(12):4640–4648, December 1982.

[58] Y. J. Wu and J. D. Fuller. An algorithm for the multiperiod market equilibrium model with geometric distributed lag demand. *Operations Research*, 44(6):1002–1012, Nov.–Dec. 1996.

[59] W. Chung, J. D. Fuller, and Y. J. Wu. A new demand-supply decomposition method for a class of economic equilibrium models. *Computational Economics*, 21:231–243, 2003.

[60] E. Çelebi and J. D. Fuller. A model for efficient consumer pricing schemes in electricity markets. *IEEE Trans. on Power Systems*, 22(1):60–67, February 2007.

[61] R. Romero and A. Monticelli. A hierarchical decomposition approach for transmission network expansion planning. *IEEE Trans on Power Systems*, 9(1):373–379, February 1994.

[62] S. Haffner, A. Monticelli, A. Garcia, J. Mantovani, and R. Romero. Branch and bound algorithm for transmission system expansion planning using a transportation model. *IEE Proc. Gener. Transm. Distrib*, 147(3):149–156, May 2000.

[63] T. S. Chung, Y. Z. Li, and Z. Y. Wang. Optimal generation expansion planning via imporved genetic algorithm approach. *Electrical Power & Energy Systems*, 26:655–659, 2004.

[64] G. Latorre, R. D. Cruz, J. M. Areiza, and A. Villegas. Classification of publications and models on transmission expansion planning. *IEEE Trans on Power Systems*, 18(2):938–946, May 2003.

[65] H. Rudnick, R. Palma, E. Cura, and C. Silva. Economically adapted transmission systems in open acces schemes - aplication of genetic algorithms. *IEEE Trans on Power Systems*, 11(3):1427–1434, August 1996.

[66] H. T. Firmo and L. F. L. Legey. Generation expansion planning: An iterative genetic algorithm approach. *IEEE Trans on Power Systems*, 17(3):901–906, August 2002.

[67] J. B. Park, Y. M. Park, J. R. Won, and K. Y. Lee. An improved genetic algorithm for generation expansion planning. *IEEE Trans on Power Systems*, 15(3):916–922, August 2000.

[68] Z. Xu and Z. Y. Dong. Market-based planning of transmission network using genetic algorithm. In *IEEE Proceedings of the* $8^{th}$ *International Conference on Probabilistic Methods Applided to Power Systems*, pages 826–831, Iowa State University, Iowa, U.S.A, 12–16 September 2004.

[69] R. A. Gallego, A. Monticelli, and R. Romero. Transmission system expansion planning by an extended genetic algorithm. *IEE Proc. Gener. Trans. Distrib.*, 145(3):329–335, May 1998.

[70] A. H. Escobar, R. A. Gallego, and R. Romero. Multistage and coordinated planning of the expansion of transmission systems. *IEEE Trans on Power Systems*, 19(2):735–744, May 2004.

[71] M. T. A. P. Wickramarathna and N. Wickramaarachchi. Transmission network planning using genetic algorithm. In *IEEE Proceedings of the 2006 Latin American Transmission & Distribution Conference and Exposition*, pages 1–5, August 2006.

[72] D. A. Pavluchenko and V. Z. Manusov. Electrical network optimization by genetic algorithm. *European Transactions on Electrical Power*, 16(6):569–576, 2006.

[73] V. Miranda, J. V. Ranito, and L. M. Proença. Genetic algorithms in optimal multistage distribution network planning. *IEEE Trans on Power Systems*, 9(4):1927–1933, November 1994.

[74] Z. Xu, Z. Y. Dong, and K. P. Wong. Transmission planning in a deregulated environment. *IEE Proc. Gener. Transm. Distrib*, 153(3):326–334, May 2006.

[75] F. Evans, J. M. Zolezzi, and H. Rudnick. Cost assignment model for electrical transmission system expansion: An approach through the kernel theory. *IEEE Trans on Power Systems*, 18(2):625–632, May 2003.

[76] N. Alguacil, A. L. Motto, and A. J. Conejo. Transmission expansion planning: A mixed-integer lp approach. *IEEE Trans on Power Systems*, 18(3):1070–1077, August 2003.

[77] F. V. Louveau and Y. Smeers. *Numerical Techniques for Stochastic Optimization*, chapter "Optimal Investments for Electricity Generation: A Stochastic Model and a Test-Problem", pages 445–453. Springer-Verlag, 1988.

[78] F. V. Louveaux. *Stochastic Programming 84*, volume II of *Mathematical Programming Studies*, chapter "Multistage Stochastic Programs with Block-Separable Recourse", pages 48–62. The Mathematical Programming Society, 1986.

[79] J. L. Higle and S. Sen. *Stochastic Decomposition A Statistical Method for Large Scale Stochastic Linear Programming*. Kluwer Academic Publishers, 1996.

[80] C. W. Therrien and M. Tummala. *Probability for Electrical and Computer Engineers*. CRC Press, 2004.

[81] R. E. Ziemer. *Elements of Engineering Probability and Statistics*. Prentice Hall, 1997.

[82] P. Kall and S.W. Wallace. *Stochastic Programming*. Wiley–Interscience Series in Systems and Optimization. John Wiley and Sons, 1994.

[83] D. P. Bertsekas. *Nonlinear Programming*. Athena Scientific, second edition, 1999.

[84] J. Nocedal and S. J. Wright. *Numerical Optimization*. Springer Series in Operations Research. Springer, 1999.

[85] J. Vajda. *Probabilistic Programming*. Probability and Mathematical Statistics. Academi Press, 1972.

[86] E. E. Sauma and S. S. Oren. Alternative economic criteria and proactive planning for transmission investment in deregulated power systems. Working Paper, August 2005. http://www.ieor.berkeley.edu/˜oren/pubs.htm.

[87] http://www.ee.washington.edu/research/pstca/.

[88] http://www-neos.mcs.anl.gov/.

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